# The Impact of a Public Option in the Health Insurance Market* 

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January 16, 2013


#### Abstract

We develop a framework where to examine the implications of the introduction of a nonprofit "public option" in the U.S. health insurance market. In this model, a continuum of heterogeneous consumers, each facing unknown medical expenditures, and differing in their expectations of such expenditures, have to choose between two competing plans. One plan is offered by a profit-maximizing private insurer; the other by social-welfare-maximizing public option. The model is calibrated based on data of U.S. medical expenditures and estimation of a Bayesian hierarchical model. The Nash Equilibrium of the resulting market structure is solved using a numerical algorithm. In equilibrium, the distinct objectives of the two insurers induce adverse selection in consumer choice: the public option covers the less healthy consumers, yielding the more profitable segment of market to the private insurer. However, our empirical results suggest that both insurers will capture significant parts of the health insurance market.


JEL Clasiffication: I11, L10, L21, L32
Keywords: Public Health Insurance; Bayesian Hierarchical Model.

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## 1 Introduction

One of the most controversial issues in the recent debate over health care reform in the United States is whether the reform should include a "public option," i.e., a non-profit insurance plan managed by the federal government that would compete with the private, for-profit insurance plans. Advocates of the public option argue that a non-profit insurance plan, with lower administrative costs and without profitability pressure, will not only provide a less expensive option to the general public, but will also discipline the private insurers because of the competition it brings to the market.

Opponents of the public option, on the other hand, warn that the public option may eventually drive out the private insurers and take over the whole insurance market. For instance, in its comments submitted to the U.S. Senate Finance Committee on June 10, 2009, the American Medical Association opposed the creation of the public option, stating that health services should be "provided through private markets, as they are currently," because "the introduction of a new public plan threatens to restrict patient choice by driving out private insurers, which currently provide coverage for nearly 70 percent of Americans," and that "the corresponding surge in public plan participation would likely lead to an explosion of costs that would need to be absorbed by taxpayers" (New York Times, June 10, 2009).

How to reform the existing health care system is apparently one of the most important challenges the United States faces at this moment when the health care expenditure is accounting for more than $16 \%$ of the GDP while continuing to increase at a faster pace than the GDP itself, and when the U.S. federal government is running an alarmingly high level of public debt. However, despite the heated debate over the legitimacy and feasibility of a "public option," there seems to be a lack of rigorous economic and quantitative analysis of the consequences of introducing such an option. For instance, the report by the Council of Economic Advisors published in 2009 (CEA 2009) mainly focuses on the impacts of overall medical reforms on long-run economic growth, employment, and government deficits, through international and states comparisons and under hypothetical scenarios, but does not provide a model-based quantitative analysis of the market equilibrium.

In this paper, we tackle this issue and estimate the potential impact of a public option in the U.S. health care system with regard to the crowd-out of private insurance from this market.

To this aim, we start by developing a theoretical model in a stochastic environment, in which a continuum of heterogeneous consumers, each facing unknown medical expenditures, but differing in their expectations of these expenditures, have to choose between two plans. One plan is offered by a profit-maximizing private insurance company; the other, by a public insurer that aims to maximize social welfare while not sustaining a budget deficit. We then estimate and calibrate the model and provide an empirical characterization of the Nash equilibrium of the market structure.

To examine the empirical characteristics of market equilibrium as implied by the model, we need to obtain a realistic calibration of the underlying distribution of the expected medical expenditures for the U.S. consumers. This is because when purchasing medical insurance, consumers are unable to observe the actual medical expenditures in the future. Rather, they have to make their decisions based on the probabilistic distributions of the actual medical expenses that they may incur over the following year. Therefore, they face uncertainty when making purchasing decisions, even with the private knowledge of their own health status.

Most of the existing studies in the literature, however, have been concentrated on analyzing a group of consumers' actual medical expenditures, for instance, by regressing the actual expenditures on various observable characteristics of the consumers in a given sample, such as age, gender, and income. Such a methodology can generate a model-based prediction of the actual expenses, i.e., a fixed number for each individual, conditional on the observed consumer characteristics. However, it is unable to capture the uncertainty or the probabilistic distribution that each individual faces when making purchasing decisions, and is only able to provide a group of different predicted values for a given set of consumers, rather than the distribution of a continuum of heterogeneous "types" or expectations of medical expenditures for the whole U.S. population. We thus decide to take a more structural approach and estimate a Bayesian hierarchical model of conjugate likelihood distribution of the expected medical expenditures. Then, we numerically solve the model and compute the market equilibrium, employing standard risk-preference parameter values and the estimated distribution of medical expenditures.

Our empirical results suggest that, in equilibrium, the public option will serve approximately two thirds of the health insurance market. The private insurer will skim the market by offering
higher deductibles and lower premiums to attract the relatively healthier consumers who do not expect high medical expenses but are mandated to purchase an insurance policy. Consequently, the private insurer will run a substantially positive profit. The public plan will take the residual part of the market, having to demand higher premiums to cover the expected higher expenditures. Thus, both the private and the insurer stay in the market despite the severe degree of adverse selection that the distinct objectives of the two insurers incur on the public option in equilibrium.

We also find that relaxing the public plan's balanced budget constraint and allowing it to run at a limited deficit will increase the public plan's market share, forcing the private plan to substantially lower its premiums and profit, but the social welfare improves. A more cost-effective public insurer, or an upper limit on the private insurer's profit margin, will also lead to a decline in the private insurer's profit and to an improvement of social welfare.

Traditionally, private entities have been the main suppliers of health insurance coverage for working age individuals in the United States. However in a few instances, the federal or state governments have intervened to improve the provision of health care to certain groups of patients. As in the current policy debate over the universal health care, concerns from the private sector have always arisen whenever the government enters the market as an alternative insurance provider, and such concerns have promoted several previous empirical studies to try to quantify the potential crowding out effect of those government programs. Yet so far researchers have not been able to reach a consensus. For instance, Cutler and Gruber (1996) estimate the effect of the Medicaid expansion to pregnant women and children on private insurance coverage, and conclude that on average $50 \%$ of the individuals who were previously covered by private insurers have switched to the new public program. ${ }^{1}$ Lo Sasso and Buchmueller (2004) obtain a similar estimate when examining the implications of the Supplemental Children's Health Insurance Program. An even larger estimate is obtained by Brown and Finkelstein (2008), in which they study the Medicaid's crowding out effect on long-term care private insurers, and conclude that even if the private insurers choose to offer comprehensive policies at actuarially fair prices, the bottom two-thirds less wealthy patients would still prefer Medicaid. On the other hand, some other studies such as Rask and Rask (2000), Lo Sasso and Meyer (2010), and Ham and Shore-Shepard (2005) suggest that the crowding

[^1]out effect of Medicaid on private insurance coverage may be very small or insignificant. A recent study by Miller and Yeo (2011) examines the potential crowding out effect in the Medicare Part D prescription drug market, and suggests that if the government plan operates at the same cost as the private insurers, it will have a negligible effect on the market structure; however, a 25 percent cost advantage would enable the government plan to capture one-fourth of the market.

In this paper, instead of focusing on some specific government programs that only target certain groups of citizens as in previous studies, we analyze and quantify the market equilibrium in the presence of a universal public health insurance plan for the whole U.S. population. Thus our study provides an alternative, more comprehensive answer to the question, and has a direct bearing on the relevant policy debates. ${ }^{2}$ Moreover, methodologically, most of the existing studies have relied on a reduced-form or statistical approach, and our paper is the first one that examines the market equilibrium in a structural game-theoretical framework. Another novelty of our study is the estimation of a Bayesian hierarchical model of a continuum of heterogeneous health "types" or expected medical expenditures for the whole U.S. population. This captures the heterogeneity of health conditions of individual consumers as well as the uncertainties that they face when purchasing insurance policies, both of which are very important characteristics of consumers in the medical insurance market in reality but have largely been ignored by existing studies.

The rest of the paper is organized as follows. Section 2 presents our theoretical model and discusses the strategy to solve and calibrate the model. Section 3 estimates a Bayesian hierarchical model to obtain the underlying distribution of expected medical expenditures. Section 4 calibrates the model and presents the empirical results. In Section 5 we discuss some modelling choices we have adopted, while Section 6 concludes.

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## 2 A Model of Health Insurance Markets

Consumers are assumed to possess a constant absolute risk aversion utility function $u(x)=-e^{-\theta x}$, with coefficient of risk aversion $\theta>0$. Each consumer incurs health expenses over the period covered by the policy that are distributed exponentially with parameter $\gamma$; the corresponding probability density function is $f(t)=\frac{1}{\gamma} e^{-\frac{t}{\gamma}}$ for $t \in[0, \infty) .{ }^{3}$ Thus, the expected health expenses of an individual of type $\gamma$ are exactly $\gamma$. The expenditure type $\gamma$ is private information of each consumer. The types are distributed in the population with cumulative distribution function $H(\cdot)$ on $[0, \infty)$, which will be estimated from the data. All consumers are mandated to purchase medical insurance.

There are two insurers in the market, a representative private insurer that maximizes expected profits, and a public insurer that aims at maximizing the expected social welfare, defined as the sum of the consumer and the producer surpluses. The public insurer must cover all consumers not covered by the private insurer, and must not run a budget deficit. In alternative specifications of the supply side of the market, we will examine situations where the non-negative profit constraint imposed on the public plan is relaxed, where the public insurer's cost-efficiency is different from that of the private insurer, and where competition within the private segment of the insurance market imposes an upper limit on the representative private insurer's profit margin.

The private insurer offers a policy characterized by two parameters $\left(p_{p}, d_{p}\right)$, where $p_{p}$ is the insurance premium and $d_{p}$ the deductible. A consumer insured by the private insurer will pay the first $d_{p}$ dollars of the realized health expenses, and the insurance plan will cover the rest. The public insurer offers an insurance policy ( $p_{g}, d_{g}$ ) with a premium $p_{g}$ and deductible $d_{g}$. In practice, insurance contracts are defined by a larger number of provisions instead of only a premium and a deductible. ${ }^{4}$ These plan characteristics serve to describe the cost and various features of a risk

[^3]sharing mechanism between insurance companies and individuals. For simplicity, we use here the deductible as a measure of the overall risk share allocated to the consumer.

The above model, while capturing the salient features of the extremely complex health insurance market, omits, for reasons of analytical and computation tractability, several additional features of this market. In Section 5 we discuss several of the simplifying modelling choices employed in this paper and their likely impact on our quantitative results. As we argue, a more comprehensive model of insurance markets will marginally improve upon the precision of our estimates, but we expect that our salient qualitative findings be robust to these richer specifications.

### 2.1 Consumer's Problem

Consider two generic insurance plans characterized by the premium-deductible pairs ( $p_{1}, d_{1}$ ) and $\left(p_{2}, d_{2}\right)$. Clearly, when $p_{i}>p_{j}$ and $d_{i}>d_{j}$, plan $\left(p_{j}, d_{j}\right)$ will dominate plan $\left(p_{i}, d_{i}\right)$ and all consumers will select the former. In the following, we assume that $d_{1}>d_{2}$ and $p_{1}<p_{2}$. Denote by

$$
\Psi(d, \theta, \gamma) \equiv \begin{cases}\frac{\theta \gamma e^{\left(\theta-\frac{1}{\gamma}\right) d}-1}{\theta \gamma-1}, & \text { when } \gamma \neq \frac{1}{\theta}  \tag{1}\\ 1+\theta d, & \text { when } \gamma=\frac{1}{\theta}\end{cases}
$$

The next proposition describes the choice of the individual of type $\gamma$ between the two contracts. Its proof is presented in Appendix A1.

Proposition 1 Assume $d_{1}>d_{2}$ and $p_{1}<p_{2}$. Then, a consumer of type $\gamma$ will choose the plan $\left(p_{1}, d_{1}\right)$ if and only if

$$
\begin{equation*}
p_{2}+\frac{1}{\theta} \ln \Psi\left(d_{2}, \theta, \gamma\right) \geq p_{1}+\frac{1}{\theta} \ln \Psi\left(d_{1}, \theta, \gamma\right) \tag{2}
\end{equation*}
$$

Thus, when evaluating a plan $(p, d)$, a consumer of type $\gamma$ trades off the flat up-front premium $p$, with an indirect premium $\frac{1}{\theta} \ln \Psi(d, \theta, \gamma)$, which is the certainty equivalent of the random prospect induced by the risk sharing mechanism, i.e., by the deductible. $\Psi(d, \theta, \gamma)$ is clearly increasing in the deductible $d$ and, as shown in appendix A2, is also increasing in the coefficient of risk aversion $\theta$ and in the type $\gamma$. The next proposition, whose proof is in Appendix A3 describes the resulting
benefits, drug formularies, exclusions, in-network providers, etc. Since these characteristics do not have clear financial implications, we do not include them in the model.
allocation of types among the two insurance plans. This separation of types is reminiscent of the insight from the seminal paper of Rothschild and Stiglitz (1976) on adverse selection in insurance markets with asymmetric information.

Proposition 2 Assume $d_{1}>d_{2}$ and $p_{1}<p_{2}$. Then, there exists a cutoff $\bar{\gamma}\left(p_{1}, d_{1}, p_{2}, d_{2}\right)$ such that a consumer of type $\gamma$ chooses plan $\left(p_{1}, d_{1}\right)$ if and only if $\gamma \leq \bar{\gamma}\left(p_{1}, d_{1}, p_{2}, d_{2}\right)$.

Thus, the healthier types, who expect low expenses, assign a lower probability to the event that the deductible will be reached, and select the plan with the lower premium and higher deductible. It can be shown that $\bar{\gamma}\left(p_{1}, d_{1}, p_{2}, d_{2}\right)$ is increasing in $p_{2}$ and $d_{2}$, and decreasing in $p_{1}$ and $d_{1}$; as expected, an increase in either the premium or the deductible of an insurance plan will make that plan less attractive, shrinking its market share.

### 2.2 Insurers' Problems

For any possible combination of the private and public plan characteristics $\left(p_{g}, d_{g}, p_{p}, d_{p}\right)$, denote the set of consumers who choose the private and public plan by $\Omega^{p}\left(p_{g}, d_{g}, p_{p}, d_{p}\right)$ and $\Omega^{g}\left(p_{g}, d_{g}, p_{p}, d_{p}\right)$, respectively. As elicited by Proposition 2, the insurer with the higher deductible and lower premium will attract the healthier individuals. For instance, when $p_{p}<p_{g}$, the private plan captures the healthier part of market and $\Omega^{p}\left(p_{g}, d_{g}, p_{p}, d_{p}\right)=\left\{\gamma: \gamma \leq \bar{\gamma}\left(p_{g}, d_{g}, p_{p}, d_{p}\right)\right\}$.

Now, given the public plan's premium and deductible ( $p_{g}, d_{g}$ ), the private insurer's decision is to choose $p_{p}$ and $d_{p}$ in order to maximize its profit $\pi^{P}\left(p_{g}, d_{g}, p_{p}, d_{p}\right)$, defined as

$$
\begin{align*}
\pi^{p}\left(p_{g}, d_{g}, p_{p}, d_{p}\right) & =\int_{\Omega^{p}\left(p_{g}, d_{g}, p_{p}, d_{p}\right)}\left(p_{p}-\int_{d_{p}}^{\infty} \frac{t-d_{p}}{\gamma} e^{-\frac{t}{\gamma}} d t\right) d H(\gamma)  \tag{3}\\
& =\int_{\Omega^{p}\left(p_{g}, d_{g}, p_{p}, d_{p}\right)}\left(p_{p}-\gamma e^{-\frac{d_{p}}{\gamma}}\right) d H(\gamma)
\end{align*}
$$

On the other hand, given the private plan's premium and deductible ( $p_{p}, d_{p}$ ), the public insurer's problem is to choose $p_{g}$ and $d_{g}$ so as to maximize the social welfare $S W\left(p_{g}, d_{g}, p_{p}, d_{p}\right)$, subject to a non-negative constraint on its profit $\pi^{g}\left(p_{g}, d_{g}, p_{p}, d_{p}\right) \geq 0$, where

$$
\begin{equation*}
\pi^{g}\left(p_{g}, d_{g}, p_{p}, d_{p}\right)=\int_{\Omega^{g}\left(p_{g}, d_{g}, p_{p}, d_{p}\right)}\left(p_{g}-\gamma e^{-\frac{d_{g}}{\gamma}}\right) d H(\gamma) \tag{4}
\end{equation*}
$$

To construct the social welfare function, we first compute the consumer surplus of an individual of type $\gamma$ who purchases a generic insurance contract $(p, d)$. For this, we compute the value of the premium $p_{0}$ that would make the individual indifferent between paying the premium $p_{0}$ for a contract with deductible $d$, and having no insurance at all. Then, the consumer surplus will be $p_{0}-p$. Since the value of the expected utility experienced by the individual with no insurance does not depend on any of the choice variables ( $p_{g}, d_{g}, p, d_{p}$ ), and thus does not affect the equilibrium game play between the two insurers, we forgo computing it and instead denote it by $U_{0}(\gamma)$. Therefore, $p_{0}$ will be the solution to the equation:

$$
\int_{0}^{d} u\left(w-p_{0}-t\right) f(t) d t+\int_{d}^{\infty} u\left(w-p_{0}-d\right) f(t) d t=U_{0}(\gamma)
$$

Solving it, we obtain

$$
p_{0}+\frac{1}{\theta} \ln \Psi(d, \theta, \gamma)=w+\frac{1}{\theta} \ln U_{0}(\gamma)
$$

Therefore,

$$
\begin{align*}
C S(\gamma, p, d) & =p_{0}-p \\
& =A(\gamma)-\frac{1}{\theta} \ln \Psi(d, \theta, \gamma)-p \tag{5}
\end{align*}
$$

where $A(\gamma)$ is a function that does not depend on $p$ or $d$. On the other hand, the expected profit that the seller of an insurance contract ( $p, d$ ) would make on an individual of type $\gamma$ is

$$
\begin{equation*}
P S(\gamma, p, d)=p-\int_{d}^{\infty} \frac{t-d}{\gamma} e^{-\frac{t}{\gamma}} d t=p-\gamma e^{-\frac{d}{\gamma}} \tag{6}
\end{equation*}
$$

Thus, from (5) and (6) it follows that

$$
\begin{align*}
S W\left(p_{g}, d_{g}, p_{p}, d_{p}\right)= & A-\int_{\Omega^{p}\left(p_{g}, d_{g}, p_{p}, d_{p}\right)}\left[\frac{1}{\theta} \ln \Psi\left(d_{p}, \theta, \gamma\right)+\gamma e^{-\frac{d}{\gamma}}\right] d H(\gamma)  \tag{7}\\
& -\int_{\Omega^{g}\left(p_{g}, d_{g}, p_{p}, d_{p}\right)}\left[\frac{1}{\theta} \ln \Psi\left(d_{g}, \theta, \gamma\right)+\gamma e^{-\frac{d_{g}}{\gamma}}\right] d H(\gamma)
\end{align*}
$$

where $A$ is a constant. Note that $S W\left(p_{g}, d_{g}, p_{p}, d_{p}\right)$ does not depend directly on the two premiums $p_{p}$ and $p_{g}$ since they represent transfers from the consumers to the insurers that do not affect the social welfare. However, the two premiums affect social welfare through their effects on the market shares of the two insurers, as elicited by $\bar{\gamma}\left(p_{g}, d_{g}, p_{p}, d_{p}\right)$. The two deductibles also affect social welfare directly through their effects on the consumer and producer surpluses associated with each
type $\gamma$. This is because a higher deductible reduces the consumer surplus, as the individual bears more of the risk, and increases the insurer's profit by reducing its expected costs.

We assume that the public and private insurers compete over market share by adjusting premiums and deductibles, and we solve the model and characterize the Nash equilibrium of the market in the empirical sections below. As we will show in section 4, in equilibrium, the private insurer will attract the healthier types leaving the public insurer to cover the remaining individuals at average cost. This feature is not generic, though, but a consequence of the particular distribution of types in the population that we estimate, and thus can only be uncovered numerically. ${ }^{5}$ Therefore, we do not impose it ex-ante but instead allow the two insurers to choose their policies freely.

### 2.3 Alternative Specifications of the Supply Side of the Market

In the baseline model presented above, we have assumed that the public option competes against a single monopolistic private insurer. Although the presence of the public insurer does induce some degree of competition in the market, the public insurer aims at maximizing social welfare instead of profit, and the private insurer is thus the only profit-maximizing player in the market. In reality, while the health insurance market does exhibit a significant and increasing degree of concentration and monopoly power, the market is not perfectly monopolistic. ${ }^{6}$ To acknowledge this in a manageable way under our model framework, we simulate an alternative scenario in which the private insurer is constrained to a profit margin that equals a fraction $\tau$ of its value in the baseline model, corresponding to the case where several monopolistic competitors in the private segment of the insurance market compete with each other and bid down their average profit margin. The parameter $\tau$ will be a measure of the competitiveness of the market.

In the later simulation analysis, we will also investigate claims that the public option may either have a stronger bargaining power with health care providers and is thus more cost effective,

[^4]or alternatively, that a lack of efficiency may actually lead to higher administrative costs of the public plan. Both of these claims have been frequently argued by both sides of the health care reform debate. We proceed by introducing an additional parameter, $\mu$, to measure the cost effectiveness of the public insurer. When $\mu=1$, the public plan is running with the same efficiency as the private plan, and the costs the public insurer faces are identical to the actual medical expenditures as drawn from the exponential distribution by individual patients. $\mu<1$ corresponds to the case that the public plan is more cost effective and can thus cover the patients at a discounted rate, and $\mu>1$ refers to the opposite case when the public insurer faces a higher operating and management cost. The private insurer always faces the same costs as the patients actually incur (i.e., as drawn from the exponential distribution).

Finally, we also explore how the market equilibrium changes if the non-negative profit constraint imposed on the public plan is relaxed, and instead of having to run a non-negative profit, the public insurer is now allowed to run a loss.

## 3 Estimation of the Underlying Distribution of Expected Medical Expenditures

We estimate the distribution of expected medical expenditures using data from the Medical Expenditure Panel Survey (MEPS), which is the most comprehensive survey on medical service of the U.S. households, their medical providers, and employers. ${ }^{7}$ The Household Component of the MEPS collects data from a sample of households in selected communities across the U.S., including information such as demographic characteristics, health conditions, use of medical services, charges and source of payments.

For our purpose, we extract the medical expenses incurred by individuals included in the dataset and estimate the underlying distribution of expected medical expenditures for the U.S. consumers.

[^5]We use the data from the year of 2007, the latest year for which the MEPS had expenditure data on when we estimated the distribution. Among the 30,964 individuals, we focus on those that are of age 64 and under, since individuals of age 65 and above are covered by the U.S. Medicare program, and are thus not the focus of the current medical policy debates or our study. Therefore our estimation sample consists of 27,238 individuals of age 64 and under in the U.S., with $35 \%$ are under the age of $18,23 \%$ are between 18 and $34,25 \%$ between 35 and 50 , and $17 \%$ between 51 and 64. The mean medical expenditure in 2007 was $\$ 2,600$, and the median $\$ 434$. Figure 1 shows the empirical histogram of the observed medical expenditures, which is heavily skewed to the right.

Since the MEPS only reports the actual or ex-post expenditures of the survey respondents but not their ex-ante expectations of medical expenditures, we need to infer the distribution of the latter from the observations of the former with the help of some statistical modeling. Because of this hierarchical structure, we estimate a Bayesian hierarchical model of conjugate likelihood distributions (George, Makov and Smith, 1993) to uncover the distribution of the individual's expectations of their own medical expenditures, $\gamma_{i}$, whose cumulative distribution function is denoted as $H(\cdot)$ as in Section 2.

We assume that $\gamma_{i}$ follows the conjugate likelihood prior distribution of a negative exponential distribution, i.e., an Inverse Gamma distribution of parameters $\alpha$ and $\beta$, which is also a highly skewed distribution to the right, similar to the histogram of the realized medical expenditures in Figure 1. Thus, conditional on $\alpha$ and $\beta$, the individual parameter distributions $\gamma_{i}$ are independent and identically distributed as

$$
\begin{equation*}
s\left(\gamma_{i} \mid \alpha, \beta\right)=\frac{\beta^{\alpha}}{\Gamma(\alpha)} \gamma_{i}^{-\alpha-1} e^{-\frac{\beta}{\gamma_{i}}} \text { for } \gamma_{i} \geq 0 \tag{8}
\end{equation*}
$$

For $\alpha$ and $\beta$ we proceed by conducting a Bayesian estimation ${ }^{8}$ by putting a prior on $\beta$ in the form of $s(\beta)=\beta^{k_{\beta 1}-1} \exp \left(-\beta / k_{\beta 2}\right)$, i.e., a Gamma distribution with parameters ( $k_{\beta 1}, k_{\beta 2}$ ), and estimating the posterior distribution of $\beta$ (given the dataset) using Markov Chain Monte Carlo (MCMC) method in a Gibbs Sampling algorithm. $\alpha$ will be estimated using a Method of Moment estimator in each round of Gibbs sampling. ${ }^{9}$ Then we will compute the median values of the

[^6]resulting distributions of $\alpha$ and $\beta$ and employ them to calibrate the model in section 2.

To use the Gibbs sampling procedure, one needs to compute analytically ( $i$ ) the posterior distribution posterior of the health type $\gamma_{i}$ given the health expense $t_{i}$ (conditional on $\alpha$ and $\beta$ ), and (ii) the posterior distribution of $\beta$ given the set of health types $\gamma_{1}, \ldots, \gamma_{n}$ (conditional on $\alpha$ ).

Proposition 3 (i) The posterior distribution posterior of the health type $\gamma_{i}$ given the health expense $t_{i}$, conditional on $\alpha$ and $\beta$, is

$$
\begin{equation*}
s\left(\gamma_{i} \mid t_{i}, \alpha, \beta\right) \propto\left(\frac{1}{\gamma_{i}} e^{-\frac{t_{i}}{\gamma_{i}}}\right)\left(\gamma_{i}^{-\alpha-1} e^{-\frac{\beta}{\gamma_{i}}}\right)=\gamma_{i}^{-\alpha-2} e^{-\frac{\beta+t_{i}}{\gamma_{i}}} \tag{9}
\end{equation*}
$$

(ii) The posterior distribution of $\beta$, given the health types $\gamma_{1}, \ldots, \gamma_{n}$, is

$$
\begin{equation*}
s\left(\beta \mid \alpha, \gamma_{1}, \ldots, \gamma_{n}\right) \propto \beta^{\alpha n+k_{\beta 1}-1} e^{-\beta\left(\sum_{i=1}^{n} \gamma_{i}^{-1}+k_{\beta 2}^{-1}\right)} \tag{10}
\end{equation*}
$$

## Proof. See Appendix A4.

Therefore, the posterior distribution of the health type $\gamma_{i}, s\left(\gamma_{i} \mid t_{i}, \alpha, \beta\right)$, is an Inverse Gamma distribution with parameters $\alpha+1$ and $\beta+t_{i}$, while the posterior of $\beta$ given the set of health types is a Gamma distribution with parameters $\alpha n+k_{\beta 1}$ and $\left(\sum_{i=1}^{n} \gamma_{i}^{-1}+k_{\beta 2}^{-1}\right)^{-1}$.

With the analytical posterior distributions of $\gamma_{i}$ and $\beta$ derived above, we are now ready to estimate the distribution $H(\cdot)$ using Markov Chain Monte Carlo (MCMC) method in a Gibbs Sampling algorithm. The parameters of the Bayesian hierarchical model to be estimated are $\alpha$ and $\beta$. To obtain the joint posterior distribution of $(\alpha, \beta)$, we simulate 100,000 iterations of the Gibbs samplers, with the first 99,000 observations as the initial burn-in period, and report the inferences of the last 1,000 random draws of the joint posterior distribution of $\alpha$ and $\beta$. For robustness we start with four different prior distributions of $\beta, s(\beta)=\operatorname{Gamma}(1,0.1) ; s(\beta)=\operatorname{Gamma}(1,1) ; s(\beta)=$ $\operatorname{Gamma}(0.1,1)$, and $s(\beta)=\operatorname{Gamma}(0.1,100)$.
posterior probability density function of $\alpha$ turns out to be quite complicated and is not a known distribution, and cannot be directly simulated. Thus an Adaptive Rejection Sampling (ARS) algorithm is adopted to numerically draw $\alpha$ in each round of the MCMC. Unfortunately, the ARS algorithm is very time-consuming, and the MCMC converges quite slowly and the results are not very stable. We finally chose to estimate $\alpha$ using a Method of Moment estimator.

In particular, for each Bayesian prior, the following steps are conducted:
1). Upon observing the actual health expense $\left\{t_{i}\right\}_{i=1}^{n}$, draw the posterior $\left\{\gamma_{i}\right\}_{i=1}^{I}$ from an Inverse Gamma distribution $\left(\alpha+1, \beta+t_{i}\right)$, as implied by (9);
2). Conditional on the simulated $\left\{\gamma_{i}\right\}_{i=1}^{I}$ and $\alpha$ obtained from last round, draw the posterior $\beta$ from a Gamma distribution $\left(\alpha n+k_{\beta 1},\left[\sum_{i=1}^{n} \gamma_{i}^{-1}+k_{\beta 2}^{-1}\right]^{-1}\right)$, as implied by (10);
3). Estimate $\alpha$ using a method-of-moments empirical Bayes argument based on $E\left(\gamma_{i}\right)=\beta /(\alpha-$ $1) \approx \sum_{i=1}^{n} \gamma_{i} / n$.

And then we go back to the step 1 and iterate the Gibbs sampler for 100,000 times.
The MCMC converges very quickly, within the first few thousands of Bayesian draws. Table 1 presents the median and $95 \%$ error bands of the marginal distributions of $\alpha$ and $\beta$ for each of the four priors. As displayed in the table, the final convergence points of the four different priors are very close, and the error bands are quite tight around the median estimates, both supporting the accuracy of the MCMC estimation. Thus the following empirical calibration of market equilibrium will be based on the median estimates of $\alpha$ and $\beta$ as reported.

Figure 2 displays the probability density function of the estimated posterior distribution of $\gamma_{i}$, with expenses normalized to units of $\$ 1,000$ in model estimation and simulation. It can be immediately seen that the estimated Inverse Gamma distribution of $\gamma_{i}$ "fits" the observed frequency of realized expenditures $t_{i}$ in Figure 1 reasonably well, and that the parameter estimates are very similar across the four different priors. For instance, the mean of the posterior distribution of $\gamma_{i}$ under prior 1 is $\$ 2,585$, compared with the mean of the actual medical expenditure of $\$ 2,600$. However, one shall note that $t_{i}$ is not the same as $\gamma_{i}$, but rather the realization of some random variable which follows an exponential distribution with a mean of $\gamma_{i}$.

## 4 Empirical Results

Based on the estimated posterior distribution of expected medical expenditures, we then solve the model for the market equilibrium. We use the NIRA (Nikaido-Isoda/Relaxation Algorithm)
as developed by Krawczyk and Zuccollo (2006) to numerically solve for a Nash Equilibrium over possible combinations of ( $p_{g}, d_{g}, p_{p}, d_{p}$ ) in the parameter space, in which the two players, the private insurer and the public insurer, choose their optimal strategies ( $p_{p}, d_{p}$ ) and ( $p_{g}, d_{g}$ ) independently, with the goal of maximizing private profit and social welfare in equations (3) and (7), respectively. With a starting point, the NIRA algorithm conducts an iterative search process for a fixed point that represents a Nash Equilibrium, and the process converges when the fixed point is obtained. ${ }^{10}$

We begin our numerical search at different starting points of $\left(p_{g}, d_{g}, p_{p}, d_{p}\right)$, and once the NIRA algorithm converges and a convergence point is obtained, we conduct a grid search around it, in order to make sure that the convergence point represents the best responses for each insurer, given the other insurer's strategy as reflected in the convergence point. Through this procedure, we can verify that the convergence point is indeed a Nash equilibrium.

On the other hand, when selecting the starting points of $\left(p_{g}, d_{g}, p_{p}, d_{p}\right)$, we use points that are evenly sparsed in the parameter space, and conduct the NIRA numerical algorithm from each of these points. Most starting points lead to non-convergence or converge to unreasonable solutions such as negative premiums or deductibles going to infinity, and are thus eliminated. The starting points that lead to convergence to a reasonable solution determine the same Nash Equilibrium for a given specification of the parameters of the model. These ensure the uniqueness of the obtained Nash equilibrium.

The model is highly non-linear and involves integrals without analytical solutions in several equations. Therefore, when solving the model and calculating each player's payoffs, a numerical quadrature algorithm is adopted to numerically integrate equations (3) and (7). ${ }^{11}$ We also employ

[^7]a non-linear equation solver to numerically solve for $\bar{\gamma}\left(p_{g}, d_{g}, p_{p}, d_{p}\right)$, the cut-off type between choosing public and private insurance plans for all possible ( $p_{g}, d_{g}, p_{p}, d_{p}$ ). Details of our numerical methodology are available upon requests.

We first calibrate the value of the CARA risk preference parameter $\theta$ to the value of $1 * 10^{-3}$ estimated by Cohen and Einav (2007) from choices of Israeli drivers for deductibles in auto insurance contracts. While Cohen and Einav (2007) present evidence that their estimated risk parameters do help predict several other coverage choices, one common concern in the literature is whether the same parameter can explain risky decisions across multiple contexts. To alleviate this concern, we also experiment with different values of $\theta$ to investigate the sensitivity of our results to the choice of $\theta$. Formal arguments (see Rabin (2000)) based on extrapolation to larger stakes suggest that decision makers should be almost risk neutral over small stakes, but empirical studies provide evidence of an inclination of individuals to over-insure against small losses. For instance, Syndor (2010) considers the case of choices of deductibles for homeowner insurance, and shows that for deductibles in the range of $\$ 100$ to $\$ 1000$, individuals paid five times more in additional premium than the actuarial value of the insurance. ${ }^{12}$ Since the stakes in Cohen and Einav (2007) are on average $\$ 100$, thus of smaller magnitude than in our case (the private insurer's deductible is approximately $\$ 500$ in our simulation exercises), we perform robustness checks with values of $\theta$ higher than the value estimated in Cohen and Einav (2007) (i.e., $2 * 10^{-3}, 5 * 10^{-3}$ and $10 * 10^{-3}$ ), as we expect that the over-insurance bias to be stronger for larger stakes.

### 4.1 Equilibrium in the Baseline Model

Based on the posterior estimates of $\alpha$ and $\beta$ and the calibrated value of $\theta$, we find the Nash Equilibrium as described above, and report in Table 2 the market equilibrium $\bar{\gamma}$, the cut-off type between choosing public and private insurance plans. The implied private and public insurers'

[^8] is divided into 1,000 sub-intervals; when calculating an integral over $(\bar{\gamma}, \infty)$, the integral is first transformed into a finite interval $(0,1)$, and the numerical quadrature is then conducted.
${ }^{12}$ As Syndor (2010) argues, the level of risk aversion elicited by the choice of the smallest of the deductibles implies that an individual should reject a gamble with an even chance of losing $\$ 1,000$ or gaining any sum of money.
premium, deductible, and market share are also reported in the table. The results corresponding to different calibration values for the risk aversion parameter $\theta$ are reported in the same table.

The results confirm that in equilibrium both insurers will capture significant parts of the market. As suggested by (7), because the public insurer maximizes social welfare instead of its profit, it is indifferent between capturing the healthier or the less healthy individuals as long as the budget constraint is satisfied. Thus, at Nash equilibrium, the private insurer will always choose to offer a lower premium and cover the healthier part of market, generating a positive profit. The less healthy consumers will enroll in the public plan, paying a higher premium but enjoying a very low deductible.

For instance, when $\theta$ is set to $1 * 10^{-3}$, the market equilibrium is characterized by a private plan charging a premium of $\$ 3,228$ and a deductible of $\$ 548$, along with a public plan with a higher premium of $\$ 3,535$ and an almost zero deductible. Even though with a higher premium, the public plan is still more attractive for consumers with an expected expenditure more than $\$ 385$, accounting for $66 \%$ of the population. Covering these relatively less healthy consumers at such premium and deductible generate zero profit for the public insurer, i.e., no fiscal burden for the government. In contrast, a lower premium combined with a higher deductible makes the private insurance plan more attractive to healthier consumers, as they expect to incur less medical expenditures during the year, and thus would prefer a plan with lower premium despite the higher deductible. The private plan will cover $34 \%$ of the whole population, generating a positive profit of $\$ 1,082$. Note also here that the deductible of the public insurer is very close to zero. This is intuitive, as subjecting individuals to risk is detrimental to the social welfare.

If the CARA parameter $\theta$ is set at $2 * 10^{-3}$, corresponding to the case when consumers are less willing to take the chance to bet on a low medical expense during the year, more consumers will choose the public plan, as the cut-off $\bar{\gamma}$ declines from $\$ 385$ to $\$ 366$ (column 2 of Table 2), and the market share of the private plan shrinks from $34 \%$ to $32 \%$. With a higher market share, the public insurer is able to offer a lower premium, at $\$ 3,440$. This is because the increased market share, i.e., those expecting an expenditure from $\$ 366$ to $\$ 385$, is composed of consumers who are healthier than the public insurer's existing pool of consumers in column 1 of Table 2, and therefore
the public insurer is able to lower the premium and still run a balanced budget. Accordingly, with a smaller market share and a lower premium, the private profit declines from $\$ 1,082$ to $\$ 987$. As we increase the CARA parameter, the public plan's market share continues to rise. For instance, as shown in columns 3 and 4 of Table 2, when the CARA parameter $\theta$ is calibrated to $5 * 10^{-3}$ and $10 * 10^{-3}$, the public plan's market share increases to $73 \%$ and $80 \%$, respectively. At the same time, the social welfare decreases as $\theta$ increases, suggesting that the decrease in social welfare determined by the additional disutility of individuals covered by the private insurer from the exposure to risk more than offsets the increase in welfare generated by the transfer of the marginal individuals from a contract with high deductible to one with almost zero deductible.

### 4.2 Equilibrium with Alternative Specifications of the Supply Side

First, we explore how the market equilibrium changes if the non-negative profit constraint imposed on the public plan is relaxed. In particular, we assume that, instead of having to run a non-negative profit, the public insurer is now allowed to run a loss, with the maximum amount of $\$ 250$, i.e., now the government needs to subsidize the public insurance plan at no more than $\$ 250$ per capita, which would amount to a maximal annual loss of $\$ 75$ billion for the whole U.S. population of 301 million in 2007 , or less than $2.7 \%$ of the U.S. federal expenditures in the same year.

Simulation results as reported in Table 3 suggest that, when the non-negative profit constraint is relaxed, the public insurer can substantially lower its premium, thus capturing a higher market share. Moreover, facing a lower public premium, the private insurer also has to lower its premium and deductible, eventually leading to a social welfare improvement. For instance, in column 1 of Table 3 where the risk-preference parameter $\theta$ is set to $1 * 10^{-3}$, the public plan is charging a premium of $\$ 2,958, \$ 577$ lower than in the same column of Table 2 when a non-negative profit constraint is imposed. The private plan responds by substantially cutting its premium and deductible as well, by $\$ 534$ and $\$ 53$, respectively. At equilibrium, the market share of the public insurer increases from $66 \%$ to $71 \%$. The private insurer's profit declines significantly as well, from $\$ 1,082$ to $\$ 769$. Social welfare improves from lower deductibles and an expansion of the public plan.

When the CARA parameter $\theta$ is set to higher levels, the public plan is capturing an even higher
market share, as shown in columns 2 to 4 and explained above. On the other hand, compared with the equilibrium under a non-negative profit constraint in Table 2, both the public and private insurers are charging substantially lower premiums - indeed the reductions in both public and private premiums substantially exceed the maximal fiscal subsidy provided by the government of $\$ 250$ per consumer. Both plans run at lower profits, but the social welfare improves.

How shall we compare the lower profitability and high premiums of the public plan with the high profitability and lower premiums of the private plan? Notice that up till now, we have assumed that both private and public insurance plans have the same management efficiency and operating costs, and the only reason why the public plan generates a lower profit at equilibrium is that, as a social welfare maximizer, the public insurer is providing insurance to the less healthy group of consumers, yielding the healthier, more profitable segment of the market to the private insurer, which turns out to run a substantial positive profit. On the other hand, as shown in Table 2, the public plan does not necessarily run on a loss. Yet providing fiscal subsidy to a limited extent will enable both the public and private insurers to substantially lower their premiums - even larger than the amount of fiscal subsidy the government provides - and improve social welfare.

A more cost-effective public insurer drives both the public and private premiums down, and increase its market share, as shown in columns 1 and 2 of Table 4. For instance, when the riskpreference parameter $\theta$ is set to $1 * 10^{-3}$, a public plan that is $5 \%$ more cost effective than before is able to lower premium by approximately $\$ 380$ and still run on a balanced budget, and occupy a larger market share of $71 \%, 5$ percentage points higher than in our benchmark case in Table 2. Facing more competitive pressure from the public insurer, the private insurer is again forced to lower its premium, from $\$ 3,223$ to $\$ 2,876$, and its profit shrinks from $\$ 1,082$ to $\$ 830$. Simulation based on a higher value of the CARA parameter generates qualitatively similar results.

On the other hand, if the public plan becomes less competitive than the private plan, for instance due to a lack of efficiency and higher administrative costs, then both the public premium and private premium increase, and public plan's market share, because it becomes less competitive, tends to decline. As shown in column 3 of Table 4, when the risk-preference parameter $\theta$ is set to $1 * 10^{-3}$, a public plan $5 \%$ less cost effective than private plan has to increase its premium to
avoid a profit loss, from $\$ 3,535$ in Table 2 to $\$ 3,914$, and the cut-off $\bar{\gamma}$ rises from $\$ 385$ to $\$ 421$, implying a significantly smaller market share of $62 \%$, compared at the $73 \%$ in Table 2. Facing a less competitive public insurer, the private insurer can now increase its premium yet still occupying a larger market share, and its profit rises from $\$ 1,082$ to $\$ 1,335$.

Finally, we investigate the scenarios in which the private segment of the insurance market faces more intense internal competition and thus the average private profit rate is lower. In particular, we impose a restriction that the private insurer's profit margin $\tau$ cannot exceed half of its equilibrium level in our benchmark case in Table 2. With this upper limit on the profit margin, the private insurer substantially lowers its deductible, by more than $50 \%$, as shown in Table 5. Facing the enhanced competition, the public plan responds by lowering its premium, and at equilibrium its market share increases slightly, by one to two percentage points. This is because, if at the equilibrium the public plan only lowers its premium to the extent that ensures the same market share as before, it runs a profit loss. Bounded by a zero-profit constraint, the public insurer is now forced to lower its premium, by more than enough to maintain the same market share as before, and ends with a slightly higher market share. With the upper limit over its profit margin, the private insurer enjoys a lower profit, yet social welfare improves significantly (Table 5). ${ }^{13}$

## 5 Discussion of Modelling Choices

In this section we discuss several of the assumptions used in constructing the stylized model of health insurance markets that we employed in this study.

On the demand side of the market, first, we assume that all individuals are required by law to purchase medical insurance, i.e., an "individual mandate", an assumption not satisfied in the current U.S. legislation, but that has been signed into law ("Patient Protection and Affordable Care Act" of March 23, 2010) and will take effect in 2014. Second, we assume that individuals purchase medical insurance by themselves, whereas in the U.S., most non-elderly people purchase

[^9]medical insurance through their employer-sponsored plans, i.e., through a two-tier decision making process in which the employers first negotiate with insurance companies and choose several plans to sponsor, and then employees make their purchase decisions within the limited set of the chosen plans, or opt out. ${ }^{14}$ However, in our model there is only one aggregated private insurer, and we thus assume that individuals make their choice entirely by themselves. ${ }^{15}$ In addition, we consider that all choices are made by consumers at individual level, rather than in clusters (for instance, in families).

Finally, another feature of the demand side of the market in our model is that individuals are assumed to have the same level of risk aversion and to be homogenous on all dimensions of their economic and social status except for their health conditions. Since consumers face lotteries over losses of relatively small magnitude (lower than $\$ 600$ ) assuming a constant absolute risk aversion (i.e., a level of risk aversion independent of the individual's wealth) is a reasonable approximation. Moreover, while some recent empirical studies have found certain degree of heterogeneity in risk aversion in insurance markets (see Cohen and Leinav (2007), Cutler, Finkelstein and McGarry (2008) and the references therein), there has not been a general consensus that we can confidently rely on when specifying our model. For instance, even the sign of the correlation between risk type and risk preference remains inconclusive in these studies. ${ }^{16}$ Because of this and of lack of data on the joint distribution of risk type and risk aversion in the market that we study, we assume in this exercise a homogeneous population on the risk preference dimension. We note however that a possible negative correlation between risk type (i.e., expected medical expenditures) and risk aversion would render the risk sharing characteristic of an insurance contract a weaker screening

[^10]tool for the private insurer as the healthier types would be more drawn to the lower deductible contracts offered by the public option. This would result in a lower market share for the private insurer. A positive correlation between the two variables would have the opposite effect.

On the supply side of the market, the main assumption in our model, made solely for computational tractability, is that each insurer offers a unique contract, and thus that there is no risk adjustment for insurance policies. The "Patient Protection and Affordable Care Act" no longer allows for risk adjustment based on pre-existing conditions or previous claim history, but it does allow for restricted pricing based on age and tobacco use. This can be easily incorporated in our current setup by performing the estimation of the underlying distribution of medical expenses for each risk group and allowing insurers to offer a contract to each of these groups. However, the additional computational burden will be substantial: because of the non-negative constraint on the public insurer's budget, the markets for different risk groups would be connected, which implies that all contracts will be chosen simultaneously in the Nash Equilibrium. We expect that the quantitative results would change when this additional institutional detail is incorporated, however the change would be marginal, since in the current Nash Equilibrium consumers are already self-selecting themselves into the two contracts according to risk types, and we do not see any a priori reason to believe that dividing consumer types into more detailed sub-types would have a significant effect on the market equilibrium on an aggregated level.

## 6 Concluding Remarks

This paper develops a stochastic game-theoretical model to analyze the consumers' behavior in choosing between a private medical insurance plan and a public insurance plan, with the former a profit-maximizer and the latter a social-welfare-maximizer who faces different profit constraints. The model is calibrated based on the data on medical expenditure from the U.S. Medical Expenditure Panel Survey and estimation of a Bayesian hierarchical model using a Markov Chain Monte Carlo (MCMC) method. The Nash Equilibrium is solved using a numerical algorithm.

Calibration results reveal that neither insurer will be completely driven out of the insurance
market, and that instead each of them will capture a significant segment of the market. At equilibrium, the public insurer will choose to cover the less healthy group of consumers, leaving the healthier, more profitable section of the market to the private insurer. Consequently, the private insurance plan generates a substantially positive profit, and the public plan runs at a balanced budget. When the risk-aversion coefficient is set to higher levels, the public plan's market share rises, the public and private insurance premiums decline, and private profit falls. We also find that when the non-negative profit constraint imposed on the public plan is relaxed, both the public and private insurers substantially lower their premiums, and the social welfare improves. A more cost-effective public insurer, or an upper limit on private insurer's profit margin, both lead to a decline in private insurer's profit and to an improvement of social welfare.

Our study constructs a framework to analyze competition between profit-maximizing private insurers and a social-welfare maximizing public insurer, and suggests that an equilibrium exists in the U.S. health insurance market where both insurers stay in the market even when the public option is subjected to the severe adverse selection (as in Rothschild and Stiglitz (1976)) that is induced by the distinct objectives of the two insurers. Empirical results of the current study provide a first-order approximation of the potential crowd-out effect of a public option in the U.S. health insurance market. Developing this framework further to incorporate additional institutional details, and thus, to refine the precision of the quantitative results obtained in this paper is an interesting avenue for future research.

## Appendix

## Appendix A1. Proof of Proposition 1

A consumer of type $\gamma$ chooses the plan $\left(p_{2}, d_{2}\right)$ if and only if

$$
\begin{gather*}
\int_{0}^{d_{1}} u\left(w-p_{1}-t\right) f(t) d t+\int_{d_{1}}^{\infty} u\left(w-p_{1}-d_{1}\right) f(t) d t \leq \\
\int_{0}^{d_{2}} u\left(w-p_{2}-t\right) f(t) d t+\int_{d_{2}}^{\infty} u\left(w-p_{2}-d_{2}\right) f(t) d t \Longleftrightarrow \\
-\int_{0}^{d_{1}} e^{-\theta\left(w-p_{1}-t\right)} \frac{1}{\gamma} e^{-\frac{t}{\gamma}} d t-\int_{d_{1}}^{\infty} e^{-\theta\left(w-p_{1}-d_{1}\right)} \frac{1}{\gamma} e^{-\frac{t}{\gamma}} d t \leq \\
-\int_{0}^{d_{2}} e^{-\theta\left(w-p_{2}-t\right)} \frac{1}{\gamma} e^{-\frac{t}{\gamma}} d t-\int_{d^{\prime}}^{\infty} e^{-\theta\left(w-p_{2}-d_{2}\right)} \frac{1}{\gamma} e^{-\frac{t}{\gamma}} d t \Longleftrightarrow \\
e^{\theta p_{1}}\left[\frac{1}{\gamma} \int_{0}^{d_{1}} e^{\left(\theta-\frac{1}{\gamma}\right) t} d t+\frac{1}{\gamma} e^{\theta d_{1}} \int_{d_{1}}^{\infty} e^{-\frac{t}{\gamma}} d t\right] \geq e^{\theta p_{2}}\left[\frac{1}{\gamma} \int_{0}^{d_{2}} e^{\left(\theta-\frac{1}{\gamma}\right) t} d t+\frac{1}{\gamma} e^{\theta d_{2}} \int_{d_{2}}^{\infty} e^{-\frac{t}{\gamma}} d t\right] \tag{11}
\end{gather*}
$$

When $\gamma \neq \frac{1}{\theta}$, equation (11) can be written as $e^{\theta p_{2}} \frac{\theta \gamma e^{\left(\theta-\frac{1}{\gamma}\right) d_{2}}-1}{\theta \gamma-1} \leq e^{\theta p_{1}} \frac{\theta \gamma e^{\left(\theta-\frac{1}{\gamma}\right) d_{1}}-1}{\theta \gamma-1}$. When $\gamma=\frac{1}{\theta}$, (11) becomes $e^{\theta p_{2}}\left(1+\theta d_{2}\right) \leq e^{\theta p_{1}}\left(1+\theta d_{1}\right)$. Using the definition of $\Psi(d, \theta, \gamma)$ from (1), these can be written concisely as $e^{\theta p_{2}} \Psi\left(d_{2}, \theta, \gamma\right) \leq e^{\theta p_{1}} \Psi\left(d_{1}, \theta, \gamma\right)$, and this completes the proof.

## Appendix A2.

We will show that $\Psi(d, \theta, \gamma)$ is increasing in $\theta$ and $\gamma$. Denote by $b(\lambda, \theta) \equiv \frac{\theta e^{(\theta-\lambda) d}-\lambda}{\theta-\lambda}$ and note that $\Psi(d, \theta, \gamma)=b\left(\frac{1}{\gamma}, \theta\right)$. We first show that $\frac{\partial}{\partial \lambda} b(\lambda, \theta)<0$ for $\lambda \in[0, \infty) \backslash\{\theta\}$. This would then immediately imply that $\frac{\partial}{\partial \gamma} \Psi(d, \theta, \gamma)=-\frac{1}{\gamma^{2}} \frac{\partial}{\partial \lambda} b\left(\frac{1}{\gamma}, \theta\right)>0$. Thus, $\frac{\partial}{\partial \lambda} b(\lambda, \theta)=\frac{-d \theta e^{(\theta-\lambda) d}(\theta-\lambda)+\theta e^{(\theta-\lambda) d}-\theta}{(\theta-\lambda)^{2}}$, for $\lambda \neq \theta$. Therefore, when $\lambda \neq \theta$, we have

$$
\begin{equation*}
\frac{\partial}{\partial \lambda} b(\lambda, \theta)<0 \Longleftrightarrow d e^{(\theta-\lambda) d}(\theta-\lambda)>e^{(\theta-\lambda) d}-1 \tag{12}
\end{equation*}
$$

Define $B_{1}(\theta) \equiv e^{(\theta-\lambda) d}$ and note that $B_{1}^{\prime}(\theta) \equiv d e^{(\theta-\lambda) d}>0$ and $B_{1}^{\prime \prime}(\theta) \equiv d^{2} e^{(\theta-\lambda) d}>0$. Thus, $B_{1}$ is increasing and strictly convex. Consider now some $\lambda<\theta$, and note that we can rewrite (12) as $d e^{(\theta-\lambda) d}>\frac{e^{(\theta-\lambda) d}-1}{\theta-\lambda}$, that is: $B_{1}^{\prime}(\theta)>\frac{B_{1}(\theta)-B_{1}(\lambda)}{\theta-\lambda}$. Since $B_{1}$ is convex and $\theta>\lambda$, this condition is satisfied. On the other hand, for $\lambda>\theta$, we rewrite (12) as $d e^{(\theta-\lambda) d}<\frac{1-e^{(\theta-\lambda) d}}{\lambda-\theta}$, that
is: $B_{1}^{\prime}(\theta)<\frac{B_{1}(\lambda)-B_{1}(\theta)}{\lambda-\theta}$. Since $B_{1}$ is convex and $\theta<\lambda$, this condition is again satisfied. Therefore, $\frac{\partial}{\partial \lambda} b(\lambda, \theta)<0$ as desired.

Next, we show that $\frac{\partial}{\partial \theta} b(\lambda, \theta)>0$ for $\theta \in[0, \infty) \backslash\{\lambda\}$, which would imply immediately $\frac{\partial}{\partial \theta} \Psi(d, \theta, \gamma)>$ 0 . We have $\frac{\partial}{\partial \theta} b(\lambda, \theta)=\frac{e^{(\theta-\lambda) d}(1+\theta d)(\theta-\lambda)-\left[\theta e^{(\theta-\lambda) d}-\lambda\right]}{(\theta-\lambda)^{2}}$, for $\theta \neq \lambda$. Denote by $B_{2}(\theta) \equiv \theta e^{(\theta-\lambda) d}$ and note that $\frac{\partial}{\partial \theta} b(\lambda, \theta)>0$ if and only if $B_{2}^{\prime}(\theta)>\frac{B_{2}(\theta)-B_{2}(\lambda)}{\theta-\lambda}$. Since $B_{2}^{\prime}(\theta) \equiv(1+\theta d) e^{(\theta-\lambda) d}>0$ and $B_{1}^{\prime \prime}(\theta) \equiv\left(2 d+\theta d^{2}\right) e^{(\theta-\lambda) d}>0$, the result follows using the same argument as for $\frac{\partial}{\partial \lambda} b(\lambda, \theta)$.

## Appendix A3. Proof of Proposition 2

By Proposition 1, a consumer of type $\gamma$ will chose the insurance plan $\left(p_{1}, d_{1}\right)$ if an only if $e^{\theta\left(p_{2}-p_{1}\right)} \geq$ $k\left(\gamma, d_{1}, d_{2}\right)$ where when

$$
k\left(\gamma, d_{1}, d_{2}\right) \equiv \frac{\Psi\left(d_{1}, \theta, \gamma\right)}{\Psi\left(d_{2}, \theta, \gamma\right)}= \begin{cases}\frac{\theta \gamma e^{\left(\theta-\frac{1}{\gamma}\right) d_{1}}-1}{}, & \text { when } \gamma \neq \frac{1}{\theta}  \tag{13}\\ \frac{1+e^{\left(\theta-\frac{1}{\gamma}\right) d_{2}}-1}{1+\theta d_{2}}, & \text { when } \gamma=\frac{1}{\theta}\end{cases}
$$

Note that $\lim _{\gamma \rightarrow \frac{1}{\theta}} k\left(\gamma, d_{1}, d_{2}\right)=k\left(\frac{1}{\theta}, d_{1}, d_{2}\right)$, so $k\left(\cdot, d_{1}, d_{2}\right)$ is continuous at $\frac{1}{\theta}$, and thus everywhere. Next, we will show that $\frac{\partial}{\partial \gamma} k\left(\gamma, d_{1}, d_{2}\right)>0$ for $\gamma \in[0, \infty) \backslash\left\{\frac{1}{\theta}\right\}$. This would complete the argument because it would imply that the condition $e^{\theta\left(p_{2}-p_{1}\right)} \leq k\left(\gamma, d_{1}, d_{2}\right)$ is satisfied for all the $\gamma \geq \bar{\gamma}\left(p_{1}, d_{1}, p_{2}, d_{2}\right)$ where $\bar{\gamma}\left(p_{1}, d_{1}, p_{2}, d_{2}\right)$ is defined implicitly by $e^{\theta\left(p_{2}-p_{1}\right)}=k\left(\bar{\gamma}, d_{1}, d_{2}\right)$.

Denote by

$$
k_{1}\left(z, \lambda_{1}, \lambda_{2}\right)=\left\{\begin{array}{c}
\frac{e^{(1-z) \lambda_{1}-z}}{e^{(1-z) \lambda_{2}-z}}, \text { when } z \neq 1 \\
\frac{1+\lambda_{1}}{1+\lambda_{2}}, \text { when } z=1
\end{array}\right.
$$

and note that $k\left(\gamma, d_{1}, d_{2}\right)=k_{1}\left(\frac{1}{\theta \gamma}, \theta d_{1}, \theta d_{2}\right)$. Thus, to show $\frac{\partial}{\partial \gamma} k\left(\gamma, d_{1}, d_{2}\right)>0$, when $d_{1}>d_{2}>0$, it is enough to show that $\frac{\partial}{\partial z} k_{1}\left(z, \lambda_{1}, \lambda_{2}\right)<0$, when $z>0$ and $\lambda_{1}>\lambda_{2}>0$. The arguments are different for the cases when $z<1$ and $z>1$, and so we will consider the two cases separately.

Lemma $4 \frac{\partial}{\partial z} k_{1}\left(z, \lambda_{1}, \lambda_{2}\right)<0$ for $z>1$ and $\lambda_{1}>\lambda_{2}>0$.

Proof. We have

$$
\frac{\partial}{\partial z} k_{1}\left(z, \lambda_{1}, \lambda_{2}\right)=\frac{\left[1+\lambda_{1} e^{(1-z) \lambda_{1}}\right]\left[z-e^{(1-z) \lambda_{2}}\right]-\left[1+\lambda_{2} e^{(1-z) \lambda_{2}}\right]\left[z-e^{(1-z) \lambda_{1}}\right]}{\left[e^{(1-z) \lambda_{2}}-z\right]^{2}}
$$

so it is enough to show that $G\left(\lambda_{1}\right)<0$ for $\lambda_{1}>\lambda_{2}>0$ where

$$
G\left(\lambda_{1}\right) \equiv\left[1+\lambda_{1} e^{(1-z) \lambda_{1}}\right]\left[z-e^{(1-z) \lambda_{2}}\right]-\left[1+\lambda_{2} e^{(1-z) \lambda_{2}}\right]\left[z-e^{(1-z) \lambda_{1}}\right]
$$

is a function of $\lambda_{1}$. Since $\left.G\left(\lambda_{1}\right)\right|_{\lambda_{1}=\lambda_{2}}=0$, it is enough to show that $G^{\prime}\left(\lambda_{1}\right)<0$ for $\lambda_{1}>\lambda_{2}$. Now, $G^{\prime}\left(\lambda_{1}\right)=z(1-z) e^{(1-z) \lambda_{1}}+e^{(1-z) \lambda_{1}}-e^{(1-z)\left(\lambda_{1}+\lambda_{2}\right)}-(1-z)\left(\lambda_{1}-\lambda_{2}\right) e^{(1-z)\left(\lambda_{1}+\lambda_{2}\right)}$, so to show $G^{\prime}\left(\lambda_{1}\right)<0$, it is enough to show that

$$
\begin{equation*}
e^{(z-1) \lambda_{2}}\left[1-z(z-1) \lambda_{1}\right]<1-\left(\lambda_{1}-\lambda_{2}\right)(z-1) \tag{14}
\end{equation*}
$$

Denote by $\operatorname{LHS}(z)$ and $R H S(z)$ the left and right hand sides of (14) as functions of $z$ and note that $\operatorname{LHS}(1)=\operatorname{RHS}(1)=1$ so it is enough to show that $L H S^{\prime}(z)<R H S^{\prime}(z)$ for $z>1$. Now,

$$
\begin{aligned}
\text { RHS }^{\prime}(z) & =\lambda_{2}-\lambda_{1} \\
\text { LHS }^{\prime}(z) & =\lambda_{2} e^{(z-1) \lambda_{2}}\left[1-z(z-1) \lambda_{1}\right]+e^{(z-1) \lambda_{2}}[-2 z+1] \lambda_{1}
\end{aligned}
$$

Note that $L H S^{\prime}(1)=R H S^{\prime}(1)=\lambda_{2}-\lambda_{1}$ so to prove that $L H S^{\prime}(z)<R H S^{\prime}(z)$ for $z>1$, it is enough to show that $L H S^{\prime \prime}(z)<R H S^{\prime \prime}(z)$ for $z>1$. We have

$$
\begin{aligned}
\operatorname{RHS}^{\prime \prime}(z)= & 0 \\
L H S^{\prime \prime}(z)= & \lambda_{2}^{2} e^{(z-1) \lambda_{2}}\left[1-z(z-1) \lambda_{1}\right]+\lambda_{2} e^{(z-1) \lambda_{2}}[-2 z+1] \lambda_{1}+\lambda_{2} e^{(z-1) \lambda_{2}}[-2 z+1] \lambda_{1}-2 e^{(z-1) \lambda_{2}} \lambda_{1} \\
= & {\left[\lambda_{2}^{2} e^{(z-1) \lambda_{2}}-\lambda_{2} z e^{(z-1) \lambda_{2}} \lambda_{1}\right]+} \\
& {\left[-\lambda_{2}^{2} e^{(z-1) \lambda_{2}} z(z-1) \lambda_{1}+\lambda_{2} e^{(z-1) \lambda_{2}}[-z+1] \lambda_{1}+\lambda_{2} e^{(z-1) \lambda_{2}}[-2 z+1] \lambda_{1}-2 e^{(z-1) \lambda_{2}} \lambda_{1}\right] }
\end{aligned}
$$

Note that $\lambda_{2}^{2} e^{(z-1) \lambda_{2}}-\lambda_{2} z e^{(z-1) \lambda_{2}} \lambda_{1}<0$ because $z>1$ and $\lambda_{2}<\lambda_{1}$. The rest of the terms in $L H S^{\prime \prime}(z)$ are all negative since $z>1$ and therefore $L H S^{\prime \prime}(z)<0=R H S^{\prime \prime}(z)$. This completes the proof of the lemma.

Lemma $5 \frac{\partial}{\partial z} k_{1}\left(z, \lambda_{1}, \lambda_{2}\right)<0$ for $0<z<1$ and $\lambda_{1}>\lambda_{2}>0$.

Proof. With a change of variable $y=-z$, we can rewrite $k_{1}\left(z, \lambda_{1}, \lambda_{2}\right)$ as $k_{2}(-z)$ where

$$
k_{2}(y) \equiv \frac{e^{(1+y) \lambda_{1}}+y}{e^{(1+y) \lambda_{2}}+y}
$$

so it is enough to show that $k_{2}^{\prime}(y)>0$ for $y \in(-1,0)$. Now,

$$
k_{2}^{\prime}(y)=\frac{\left[\lambda_{1} e^{(1+y) \lambda_{1}}+1\right]\left[e^{(1+y) \lambda_{2}}+y\right]-\left[\lambda_{2} e^{(1+y) \lambda_{2}}+1\right]\left[e^{(1+y) \lambda_{1}}+y\right]}{\left[e^{(1+y) \lambda_{2}}+y\right]^{2}}
$$

so it is enough to show that $J(y)>0$ for $y \in(-1,0)$ where

$$
\begin{aligned}
J(y) & \equiv\left[\lambda_{1} e^{(1+y) \lambda_{1}}+1\right]\left[e^{(1+y) \lambda_{2}}+y\right]-\left[\lambda_{2} e^{(1+y) \lambda_{2}}+1\right]\left[e^{(1+y) \lambda_{1}}+y\right] \\
& =\left(\lambda_{1}-\lambda_{2}\right) e^{(1+y)\left(\lambda_{1}+\lambda_{2}\right)}+y \lambda_{1} e^{(1+y) \lambda_{1}}-y \lambda_{2} e^{(1+y) \lambda_{2}}+e^{(1+y) \lambda_{2}}-e^{(1+y) \lambda_{1}}
\end{aligned}
$$

$J(-1)=0$, so it is enough to show that $J^{\prime}(y)>0$ for $y \in(-1,0)$. But $J^{\prime}(y)=\left(\lambda_{1}^{2}-\lambda_{2}^{2}\right) e^{(1+y)\left(\lambda_{1}+\lambda_{2}\right)}+$ $y \lambda_{1}^{2} e^{(1+y) \lambda_{1}}-y \lambda_{2}^{2} e^{(1+y) \lambda_{2}}$. By writing $J^{\prime}(y)$ as a function of $\lambda_{1}$, we denote by

$$
I\left(\lambda_{1}\right) \equiv J^{\prime}(y)=\left(\lambda_{1}^{2}-\lambda_{2}^{2}\right) e^{(1+y)\left(\lambda_{1}+\lambda_{2}\right)}+y \lambda_{1}^{2} e^{(1+y) \lambda_{1}}-y \lambda_{2}^{2} e^{(1+y) \lambda_{2}}
$$

Note that $\left.I\left(\lambda_{1}\right)\right|_{\lambda_{1}=\lambda_{2}}=0$, so it is enough to show that $I^{\prime}\left(\lambda_{1}\right)>0$ for $\lambda_{1}>\lambda_{2}$. But $I^{\prime}\left(\lambda_{1}\right)=$ $2 \lambda_{1} e^{(1+y)\left(\lambda_{1}+\lambda_{2}\right)}+\left(\lambda_{1}^{2}-\lambda_{2}^{2}\right)(1+y) e^{(1+y)\left(\lambda_{1}+\lambda_{2}\right)}+2 y \lambda_{1} e^{(1+y) \lambda_{1}}+y(1+y) \lambda_{1}^{2} e^{(1+y) \lambda_{1}}$, so it is enough to show that

$$
\begin{equation*}
\left[2 \lambda_{1}+\left(\lambda_{1}^{2}-\lambda_{2}^{2}\right)(1+y)\right] e^{(1+y) \lambda_{2}}>(-y)\left[2 \lambda_{1}+(1+y) \lambda_{1}^{2}\right] \tag{15}
\end{equation*}
$$

Denote by $L H S(y)$ and $R H S(y)$ the left and right hand sides of (15) and note that $L H S(-1)=$ $R H S(-1)=2 \lambda_{1}$. Therefore, to prove the inequality, it is enough to show that $L H S^{\prime}(y)>R H S^{\prime}(y)$ for $y \in(-1,0)$. Now, $L H S^{\prime}(y)=\left[2 \lambda_{1} \lambda_{2}-\lambda_{2}^{2}+\left(\lambda_{1}^{2}-\lambda_{2}^{2}\right)(1+y) \lambda_{2}+\lambda_{1}^{2}\right] e^{(1+y) \lambda_{2}}$ and $R H S^{\prime}(y)=$ $-2 \lambda_{1}-(2 y+1) \lambda_{1}^{2}$. Note that, $2 \lambda_{1} \lambda_{2}>\lambda_{2}^{2}$ because $\lambda_{1}>\lambda_{2},\left(\lambda_{1}^{2}-\lambda_{2}^{2}\right)(1+y) \lambda_{2}>0$ because $1+y>0, \lambda_{1}^{2}>-(2 y+1) \lambda_{1}^{2}$ because $y \in(-1,0)$ and $e^{(1+y) \lambda_{2}}>1$ because $1+y>0$. Therefore, indeed $L H S^{\prime}(y)>R H S^{\prime}(y)$ for $y \in(-1,0)$ which completes the proof of the lemma.

This completes the proof of proposition 2.

## Appendix A4. Proof of Proposition 3

(i) Conditional on $\alpha$ and $\beta$, the posterior for the individual $i$ 's health type $\gamma_{i}$ upon observing the individual's health expenses $t_{i}$ is

$$
s\left(\gamma_{i} \mid t_{i}, \alpha, \beta\right)=\frac{s\left(\gamma_{i}, t_{i} \mid \alpha, \beta\right)}{s\left(t_{i} \mid \alpha, \beta\right)}=\frac{s\left(t_{i} \mid \gamma_{i}, \alpha, \beta\right) s\left(\gamma_{i} \mid \alpha, \beta\right)}{s\left(t_{i} \mid \alpha, \beta\right)}
$$

The result in (9) follows immediately from $s\left(t_{i} \mid \gamma_{i}, \alpha, \beta\right)=\frac{1}{\gamma_{i}} e^{-\frac{t_{i}}{\gamma_{i}}}$ and $s\left(\gamma_{i} \mid \alpha, \beta\right)=\frac{\beta^{\alpha}}{\Gamma(\alpha)} \gamma_{i}^{-\alpha-1} e^{-\frac{\beta}{\gamma_{i}}}$.
(ii) As a first step, we will show that the likelihood $L\left(\alpha, \beta \mid \gamma_{1}, \ldots, \gamma_{n}\right)$ is integrable and logconcave.

Lemma 6 The likelihood $L\left(\alpha, \beta \mid \gamma_{1}, \ldots, \gamma_{n}\right)$ is integrable and log-concave in $(\alpha, \beta)$ for any $n \geq 2$.

Proof. Note that $f(t \mid \gamma)=\frac{1}{\gamma} e^{-\frac{t}{\gamma}} I_{[0, \infty)}(t)$ can be written as

$$
f(t \mid \gamma)=e^{t \gamma^{*}-\Psi\left(\gamma^{*}\right)} d \nu(t)
$$

where $\gamma^{*} \equiv-\frac{1}{\gamma}, \Psi\left(\gamma^{*}\right) \equiv-\ln \left(-\gamma^{*}\right)$ and $d \nu(t) \equiv I_{[0, \infty)}(t)$.
On the other hand, $s(\gamma \mid \alpha, \beta)=\frac{\beta^{\alpha}}{\Gamma(\alpha)} \gamma^{-\alpha-1} e^{-\frac{\beta}{\gamma}} I_{(0, \infty)}(\gamma)$ can be written as

$$
s(\gamma \mid \alpha, \beta)=e^{x_{0} \gamma^{*}-m_{0} \Psi\left(\gamma^{*}\right)-\varphi\left(x_{0}, m_{0}\right)} I_{\Gamma^{*}}(\gamma)
$$

where $x_{0} \equiv \beta, m_{0} \equiv \alpha+1, \varphi\left(x_{0}, m_{0}\right) \equiv \ln \Gamma\left(m_{0}-1\right)-\left(m_{0}-1\right) \ln x_{0}$ and $\Gamma^{*} \equiv(-\infty, 0)$. Since $\alpha$ and $\beta$ are linear functions of $x_{0}$ and $m_{0}$, the claim follows from Theorem 3.3 in George, Makov and Smith (1993).

Since the prior $s(\alpha, \beta)$ is also log-concave and integrable, from Lemma 6 it follows that the posterior $s\left(\alpha, \beta \mid \gamma_{1}, \ldots, \gamma_{n}\right)$ will be a proper distribution. Therefore, since $s\left(\alpha, \beta \mid \gamma_{1}, \ldots, \gamma_{n}\right)=$ $L\left(\alpha, \beta \mid \gamma_{1}, \ldots, \gamma_{n}\right) s(\alpha, \beta)$, assuming separability of the prior, we have:

$$
\begin{aligned}
s\left(\beta \mid \gamma_{1}, \ldots, \gamma_{n}, \alpha\right) & =\frac{s\left(\alpha, \beta \mid \gamma_{1}, \ldots, \gamma_{n}\right)}{s\left(\alpha \mid \gamma_{1}, \ldots, \gamma_{n}\right)}= \\
& \propto \frac{L\left(\alpha, \beta \mid \gamma_{1}, \ldots, \gamma_{n}\right) s(\alpha, \beta)}{s\left(\alpha \mid \gamma_{1}, \ldots, \gamma_{n}\right)} \\
& =L\left(\alpha, \beta \mid \gamma_{1}, \ldots, \gamma_{n}\right) s(\beta) k_{1}(\alpha)
\end{aligned}
$$

for some function $k_{1}(\alpha)$. Since the likelihood function of an Inverse Gamma distribution is

$$
\begin{equation*}
L\left(\alpha, \beta \mid \gamma_{1}, \ldots, \gamma_{n}\right) \propto\left[\frac{\beta^{\alpha}}{\Gamma(\alpha)}\right]^{n}\left(\prod_{i=1}^{n} \gamma_{i}^{-1}\right)^{\alpha+1} e^{-\beta \sum_{i=1}^{n} \gamma_{i}^{-1}} \tag{16}
\end{equation*}
$$

it follows that

$$
s\left(\beta \mid \gamma_{1}, \ldots, \gamma_{n}, \alpha\right) \propto\left[\frac{\beta^{\alpha}}{\Gamma(\alpha)}\right]^{n}\left(\prod_{i=1}^{n} \gamma_{i}^{-1}\right)^{\alpha+1} e^{-\beta \sum_{i=1}^{n} \gamma_{i}^{-1}} s(\beta) k_{2}(\alpha)
$$

Collecting the terms that depend on $\beta$, we have:

$$
s\left(\beta \mid \gamma_{1}, \ldots, \gamma_{n}, \alpha\right) \propto \beta^{\alpha n} e^{-\beta \sum_{i=1}^{n} \gamma_{i}^{-1}} s(\beta)
$$

which, since the prior $s(\beta)=\beta^{k_{\beta 1}-1} \exp \left(-\beta / k_{\beta 2}\right)$, proves (10).

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Table 1: Posterior Distribution of $\gamma_{i}$

|  | Prior 1 | Prior 2 | Prior 3 | Prior 4 |
| :--- | :---: | :---: | :---: | :---: |
| $\alpha$ | 1.1956 | 1.1944 | 1.1949 | 1.1954 |
| Median | $(1.1778,1.2063)$ | $(1.1772,1.2066)$ | $(1.1780,1.2052)$ | $(1.1802,1.2066)$ |
| $95 \%$ band |  |  |  |  |
| $\beta$ | 0.5056 | 0.5032 | 0.5044 | 0.5055 |
| Median | $(0.4917,0.5186)$ | $(0.4893,0.5178)$ | $(0.4904,0.5170)$ | $(0.4926,0.5193)$ |
| $95 \%$ band |  |  |  |  |

Table 2: Nash Equilibrium with A Non-negative Public Profit Constraint

|  | $\theta=1 * 10^{-3}$ | $\theta=2 * 10^{-3}$ | $\theta=5 * 10^{-3}$ | $\theta=10 * 10^{-3}$ |
| :--- | ---: | ---: | ---: | ---: |
| Public Plan |  |  |  |  |
| Premium | $\$ 3,535$ | $\$ 3,440$ | $\$ 3,234$ | $\$ 2,988$ |
| Deductible | $\$ 4$ | $\$ 3$ | $\$ 15$ | $\$ 1$ |
| Profit | $\$ 0$ | $\$ 0$ | $\$ 0$ | $\$ 0$ |
|  |  |  |  |  |
| Private Plan |  | $\$ 3,228$ | $\$ 3,133$ | $\$ 2,937$ |
| Premium | $\$ 548$ | $\$ 520$ | $\$ 465$ | $\$ 2,613$ |
| Deductible | $\$ 1,082$ | $\$ 987$ | $\$ 792$ | $\$ 525$ |
| Profit |  |  |  | $\$ 527$ |
|  |  | $\$ 366$ | $\$ 326$ |  |
| Market Share | $\$ 385$ | $68.0 \%$ | $72.5 \%$ | $79.5 \%$ |
| $\gamma^{*} * 1000$ | $65.9 \%$ | $32.0 \%$ | $27.5 \%$ | $20.5 \%$ |
| Public Share |  |  |  |  |
| Private Share | $34.1 \%$ |  |  | $-\$ 2,430.4$ |
| Social Welfare | $-\$ 2,419.7$ | $-\$ 2,423.4$ | $-\$ 2,441.6$ |  |
|  |  |  |  |  |

Table 3: Nash Equilibrium with a Maximal Public Loss of \$250

|  | $\theta=1 * 10^{-3}$ | $\theta=2 * 10^{-3}$ | $\theta=5 * 10^{-3}$ | $\theta=10 * 10^{-3}$ |
| :--- | ---: | ---: | ---: | ---: |
| Public Plan |  |  |  |  |
| Premium | $\$ 2,958$ | $\$ 2,900$ | $\$ 2,755$ | $\$ 2,274$ |
| Deductible | $\$ 6$ | $\$ 3$ | $\$ 6$ | $\$ 5$ |
| Profit | $-\$ 250$ | $\$-250$ | $-\$ 250$ | $-\$ 249$ |
|  |  |  |  |  |
| Private Plan | $\$ 2,689$ | $\$ 2,625$ | $\$ 2,473$ | $\$ 2,020$ |
| Premium | $\$ 495$ | $\$ 482$ | $\$ 445$ | $\$ 483$ |
| Deductible | $\$ 769$ | $\$ 708$ | $\$ 563$ | $\$ 102$ |
| Profit |  |  |  |  |
|  | $\$ 339$ | $\$ 325$ | $\$ 292$ | $\$ 151.4$ |
| Market Share | $70.9 \%$ | $72.5 \%$ | $76.8 \%$ | $94.8 \%$ |
| $\gamma^{*} * 1000$ | $29.1 \%$ | $27.5 \%$ | $23.2 \%$ | $5.2 \%$ |
| Public Share |  |  |  |  |
| Private Share | $-\$ 2,413.8$ | $-\$ 2,421.2$ | $-\$ 2,426.9$ | $-\$ 2,419.9$ |
| Social Welfare |  |  |  |  |

Table 4: Nash Equilibrium with Alternative Cost Effectiveness of Public Option

|  | $\mu=0.95$ |  | $\mu=1.05$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\theta=1 * 10^{-3}$ | $\theta=10 * 10^{-3}$ | $\theta=1 * 10^{-3}$ | $\theta=10 * 10^{-3}$ |
| Public Plan |  |  |  |  |
| Premium | \$3,156 | \$2,734 | \$3,914 | \$3,259 |
| Deductible | \$4 | \$0 | \$1 | \$5 |
| Profit | \$0 | \$0 | \$0 | \$0 |
| Private Plan |  |  |  |  |
| Premium | \$2,876 | \$2,403 | \$3,584 | \$2,860 |
| Deductible | \$525 | \$475 | \$562 | \$550 |
| Profit | \$830 | \$404 | \$1,335 | \$672 |
| Market Share |  |  |  |  |
| $\gamma^{*} * 1000$ | \$342 | \$245 | \$421 | \$297 |
| Public Share | 70.7\% | 82.9\% | 62.2\% | 76.1\% |
| Private Share | 29.3\% | 17.1\% | 37.8\% | 23.9\% |
| Social Welfare | -\$2,307.2 | -\$2,320.4 | -\$2,541.8 | -\$2,572.7 |

Table 5: Nash Equilibrium with an Upper Limit on Private Insurers' Profit Margin

|  | $\theta=1 * 10^{-3}$ | $\theta=2 * 10^{-3}$ | $\theta=5 * 10^{-3}$ | $\theta=10 * 10^{-3}$ |
| :--- | ---: | ---: | ---: | ---: |
| Public Plan |  |  |  |  |
| Premium | $\$ 3,490$ | $\$ 3,362$ | $\$ 3,149$ | $\$ 2,814$ |
| Deductible | $\$ 0$ | $\$ 0$ | $\$ 0$ | $\$ 0$ |
| Profit | $\$ 0$ | $\$ 0$ | $\$ 0$ | $\$ 0$ |
| Private Plan |  |  |  |  |
| Premium | $\$ 3,385$ | $\$ 3,253$ | $\$ 3,022$ | $\$ 2,694$ |
| Deductible | $\$ 123$ | $\$ 129$ | $\$ 155$ | $\$ 148$ |
| Profit | $\$ 1,064$ | $\$ 935$ | $\$ 722$ | $\$ 390$ |
|  |  |  |  |  |
| Market Share |  | $\$ 350$ | $\$ 304$ | $\$ 230$ |
| $\gamma^{*} * 1000$ | $\$ 375$ | $69.8 \%$ | $75.0 \%$ | $84.9 \%$ |
| Public Share | $64.9 \%$ | $30.2 \%$ | $25.0 \%$ | $15.1 \%$ |
| Private Share | $33.1 \%$ |  |  |  |
| Social Welfare | $-\$ 2,415.0$ | $\$-2,415.3$ | $-\$ 2,416.5$ | $-\$-2,416.9$ |
|  |  |  |  |  |

Figure 1. Sample from the U.S. Medical Expenditure Panel Survey


Figure 2. Posterior Distribution of Expected Medical Expenditure



[^0]:    *We would like to thank Benedicte Apouey, Daniel Miller, Sorin Maruster and Christian Roessler for helpful comments, and to Gabriel Picone for feedback throughout the work on this project. We are also very grateful to Jacek Krawczyk for sharing with us the NIRA software.
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[^1]:    ${ }^{1}$ When revisited 10 years later, Gruber and Simon (2008) obtain an even higher estimate at $60 \%$.

[^2]:    ${ }^{2}$ Our framework can also be employed to examine competition between for-profit and public insurers in other markets, such as the insurance market for individuals of age 65 and above, if policy makers decide to replace Medicare with a premium support program and allow private insurers to compete with the traditional Medicare.

[^3]:    ${ }^{3}$ The exponential distribution, which we employ for its analytical tractability and versatility, is a particular case of the Weibull distribution with shape parameter equal to 1 . These distributions are employed in the literature to model the health expenses distribution with a parametrized hazard function that describes the likelihood of any possible claim level. See Basu, Manning and Mullahy (2004) for an overview of these models.
    ${ }^{4}$ These provisions may include coinsurance rates, copayments, etc. In addition, current health insurance plans may also specify annual or lifetime coverage limits. According to the new health care law, insurers are no longer allowed to include coverage limits in the design of their plans. Finally, health insurance plans may specify covered

[^4]:    ${ }^{5}$ It can be shown by counterexample that there exist distributions of health types for which the private insurer chooses to attract the less healthy individuals because they constitute a larger fraction of the population, and thus allow for higher revenues even if at a higher cost.
    ${ }^{6}$ Dafny (2010) elicits the presence of market power in most insurance markets by identifying increases in premiums for employer-sponsored plans following a positive profitability shock to an employer.

[^5]:    ${ }^{7}$ Carlin and Town (2009), Abaluck and Gruber (2011) or Handel (2012) employ cost models applied to employerlevel datasets of individual choices of health insurance plans. Our cost model is estimated on the whole U.S. population under the age of 65 .

[^6]:    ${ }^{8}$ Rather than treating $\alpha$ and $\beta$ as fixed, as in Gaver \& O'Muircheartaigh (1987), we adopt a strategy similar to the one in Gelfand and Smith (1990).
    ${ }^{9}$ We have also experimented a full-scale Bayesian estimation by putting an exponential prior on $\alpha$ as well. The

[^7]:    ${ }^{10}$ The NIRA procedure is based on the observation that the problem of finding the fixed point of a correspondence is equivalent to that of maximizing a properly defined induced function. Thus, using the standard notation from a normal form game of a player $i$ 's strategy by $s_{i} \in S_{i}$, and of the resulting payoffs $u_{i}: \mathbf{S} \rightarrow \mathbb{R}$ by $u_{i}(\mathbf{s})$, one defines the Nikaido-Isoda function $\Psi: \mathbf{S} \times \mathbf{S} \rightarrow \mathbb{R}$ by $\Psi\left(\mathbf{s}^{\prime}, \mathbf{s}\right) \equiv \sum_{i=1}^{n}\left[u_{i}\left(s_{i}^{\prime}, s_{-i}\right)-u_{i}\left(s_{i}, s_{-i}\right)\right]$. It is straightforward to see that $\Psi(\mathbf{s}, \mathbf{s})=0$ and that $\Psi_{0}(\mathbf{s}) \equiv \max _{\mathbf{s}^{\prime} \in \mathbf{S}} \Psi\left(\mathbf{s}^{\prime}, \mathbf{s}\right) \geq 0$ for all $\mathbf{s} \in \mathbf{S}$. Moreover, $\mathbf{s}$ is a Nash equilibrium of the game if and only if $\Psi_{0}(\mathbf{s})=0$. When the set of available actions for a player is endogenously defined by the actions of the other players according to some constraint function, the Nikaido-Isoda function is defined in terms of the resulting Lagrangian that includes the constraint function. For more details, see Krawczyk and Zuccollo (2006).
    ${ }^{11}$ To ensure the accuracy of numerical integration, we adopt a composite trapezoidal quadrature rule and divide

[^8]:    the integral interval at a very refined level, for instance, when calculating an insurer's profit over $(0, \bar{\gamma})$, the interval

[^9]:    ${ }^{13}$ The uniqueness of the Nash Equilibrium continues to hold when the cap on the private insurer's profit is imposed. There may exist multiple contracts that the private insurer can offer to attain that cap, but the zero budget constraint of the public insurer ensures that there exists only one fixed point of the two insurers' best responses.

[^10]:    ${ }^{14}$ According to a survey conducted by Kaiser Family Foundation (2010), individual, or non-group, health insurance covers about 14 million nonelderly people in the U.S., making it the least common source of health insurance. In contrast, about 157 million nonelderly people are covered by employer-sponsored insurance.
    ${ }^{15}$ Moreover, whether employer-based health insurance should continue to be the format going forward is again up for debate (Reinhardt 2009), and our simulation study based on this assumption is still very relevant.
    ${ }^{16}$ For instance, Cohen and Leinav (2007) unveil a positive relationship in the choice of deductibles in auto insurance contracts, while Cutler, Finkelstein and McGarry (2008) obtain a negative one in different insurance markets. In the context of health insurance, a negative correlation between risk type and risk preference, i.e., a case where healthier individuals are more risk averse, could be explained by heathier behavioral and lifestyle choices of more risk averse individuals. On the other hand, a positive relationship could emerge if less healthy individuals' past negative experiences are conducive of more aversion towards risk.

