The Patent-issuing Rules and Economic Growth: Are We in a "Wrong" Patent Regime?*

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Abstract

Probably, yes.

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1 Introduction

Modern economic growth theory emphasizes technological progress as the engine of growth. In this paper, we study how a country's patent-issuing policy can affect innovation and economic growth.

When there are competing claimants to the same innovation, the patent goes to the one who filed a patent application first. While this simple first-to-file rule is the norm everywhere in the world today, it was not so in the United States before the enactment of the 2012 America-Invents Act. Earlier, the U.S. had instead adhered to an alternative first-to-invent rule, by which it granted the patent to the one who claimed to have discovered the innovation first. Previous attempts to switch to a first-to-file rule had been met with strong opposition. Opponents had argued that a switchover would prove detrimental to America's inventiveness, adducing the fact that the U.S. had led the world in invention for more than a century thanks to the first-to-invent feature of its patent law in effect since 1836. This argument, if valid, implies that the world economy is in the "wrong" patent regime. The objective of this paper is to investigate which of the two rules generates faster economic growth.

The question of which rule is more conducive to innovation was first investigated by Scotchmer and Green (1990) in a non-growth context. They considered a two-stage R&D race between two firms, in which firms must discover the intermediate and the final innovation successively to bring a new product to market. They argued that a firm is more likely to patent the intermediate innovation under a first-to-file rule lest it be claimed by the rival who discovers it later. A patent puts the intermediate innovation in the public domain and makes the second-stage competition a two-firm race. Since two firms can make a discovery faster than one, it was concluded that first-to-file is more conducive to innovation than first-to-invent.

The Scotchmer-Green result, however, proves to be sensitive to the key assumptions of the model. Miyagiwa and Ohno (2015) found that their result can be reversed when innovation probabilities depend on firms' R&D efforts. In a different setting, Miyagiwa (2015) showed that a reversal can also occur when a firm remains ignorant of a rival's discovery unless it is patented.

All these works focused on innovation of a single product in partial-equilibrium settings. Although the approach generates interesting insights, it is not certain whether partial-equilibrium analyses can correctly predict the effects on economic growth, an inherently aggregate phenomenon. Furthermore, opponents in the U.S. seem to have been more concerned with the long-run effect of adopting a first-to-file rule. The objective of this paper is thus to examine the effect of patent-issuing rules on economic growth in a general equilibrium model.

To model economic growth, we draw on Romer (1990) and consider an economy with three sectors. The R&D sector invents blueprints for intermediate goods. The intermediate-goods sector buys the blueprints to produce specialized intermediate goods under monopolistic competition. The final good sector turns all the existing intermediate goods into a single consumption good. Labor, the only factor of production, is employed in the R&D sector and the intermediate goods sector. The economy grows as new varieties of intermediate goods are created. The steady-state growth rates are computed and compared under the first-to-file and the first-to-invent rule.

Although the basic setup is standard, our analysis has two novel features. The first and foremost is in the R&D sector. Here, we remain as close as possible to the Scotchmer-Green setup. Namely, we assume the following.

1. Two firms engage in two-stage R&D for the invention of each intermediate good.
2. R&D is sequential; to invent a blueprint for each intermediate good, firms must discover the first-stage innovation before proceeding to the second stage of R&D.
3. Innovation probabilities are exogenous.
4. The first-stage innovation and the second-stage innovation are independently patentable.
5. Information is complete, i.e. when a firm has discovered the first-stage innovation, the rival knows it, but does not learn its content unless it is patented.

These assumptions ensure that the Scotchmer-Green result is not reversed by other factors noted in the precursory studies, and allow us to zero in on the general-equilibrium growth-theoretic aspects of the patent-issuing rules.

The second novel feature of our model is the introduction of asymmetry in the invention of intermediate goods across industries. While asymmetry can be introduced in a number of ways, we focus on the R&D cost asymmetry. In such a setting, the two patent-issuing rules affect each industry's incentive to patent the first-stage innovations differently, generating a differential impact on resource allocation and economic growth.

Our analysis proceeds in two steps. We first extend the Scotchmer-Green model to a continuum of industries with asymmetric R&D costs. It is still a partial-equilibrium model because the profits and wages are taken as given. In this "basic" model, first-to-file induces faster growth in product development than first-to-invent. This is the growth-theoretic version of the Scotchmer-Green result.

The second step of our analysis extends the basic model to a general-equilibrium framework, where intertemporal utility maximization and intersectoral labor mobility jointly determine consumer expenditure and profits (the prize for innovation) in terms of the wage, the numeraire.

In contrast to the partial equilibrium model, the general equilibrium model gives rise to several possible cases to analyze. To select a most plausible and data-consistent equilibrium, we restrict our attention to cases in which a more patient economy grows faster. This criterion is intuitive, empirically supported (e.g., Dohmen (2016) and Hübner, M. and G. Vannoorenberghe (2015)), and widely observed in the standard endogenous growth models (e.g., Romer (1990) and Lucas (1998)). Under this equilibrium-selection criterion, our analysis shows that first-to-invent promotes faster economic growth than first-to-file, reversing the partial-equilibrium result. The reversal occurs for the following reason. First, in general equilibrium, profits are lower in first-to-invent than in first-to-file. Second, lower profits make patenting more attractive, inducing more industries to patent innovations. Third, patents disclose information and lead to faster economic growth. The analysis below explains these linkages in detail.

We now discuss this paper's main contributions in light of relevant literature. They are twofold. One is to the line of research in the industrial-organization literature. The works cited earlier examine which patent system is more conducive to innovation of a single product, but leave unanswered the question of which patent system promotes faster economic growth in an aggregate economy. Thus, this paper is a natural extension of the precursory studies of innovation to a multi-industry model.

Our second contribution is to the literature on economic growth theory. Romer (1990) and Aghion and Howitt (1992) pioneered in developing the growth models based on expanding product variety and rising product quality, respectively. Grossman and Helpman (1991) applied these two approaches to international trade models. These seminal works have spawned much research on the linkages among growth, innovation, and trade. Importantly, the above models have offered a tractable framework in which the effect of patent policy can be examined. For example, the effects of a stronger patent protection can be studied, and optimal patent length can be characterized (e.g., Grossman and Lai (2004)). However, this body of literature assumes one-stage R&D, that is, a patent is immediately granted once a single innovation occurs. Thus, the mechanisms through which firms' *strategic* patenting affects long-run growth remains unexplored.[1] Our paper fills this lacuna. We believe this to be a particularly important contribution of the present paper because patents are widely considered a key policy tool to promote long-run economic growth.

Turning to empirical studies on the issue of first-to-file versus first-to-invent, there is only a scanty literature. Lo and Sutthiphisal (2009) used Canada's 1989 decision to switch from first-to-invent to first-to-file as a natural experiment and found that the switchover failed to stimulate Canadian R&D efforts. Abrams and Wagner (2012), also using Canadian data, discovered significant drops in patent applications from individual and small-scale corporate inventors. These results are consistent with our key result that first-to-file is less conducive to technical progress relative to first-to-invent.

The remainder of the paper is organized in 5 sections. Section 2 describes the R&D sector, which is the core of the model. The section finds what induces firms to patent innovations in first-to-file and first-to-invent. Section 3 compares the growth rates under the two patent regimes in a partial equilibrium setting where the prize for innovation is exogenously given. Section 4 extends the analysis of section 3 to general equilibrium. It is demonstrated that the Scotchmer-Green result is reversed once the prize of innovation is endogenously determined. Section 5 concludes.

2 The R&D Sector

We begin with the R&D sector, which is the most salient component of our model. The production sectors and the growth mechanism will be taken up in the next section.

Assume a unit mass of intermediate-goods industries indexed by $j \in [0, 1]$. Industry j produces $n_j(t)$ varieties of differentiated intermediate goods at time t. R&D is conducted in all industries with the same key characteristics as in the Scotchmer-Green model. First, a blueprint for each intermediate good is developed through two-stage R&D competition between two rival R&D firms. The first-stage innovation is labeled by A and the second-stage innovation by B. Second, R&D is sequential. To create a complete blueprint for a new variety, firms must first discover A and then B. Third, R&D is stochastic; A is discovered with hazard rate α and B with hazard rate β. These hazard rates are exogenous and common in all industries.[2] Fourth, both innovations, A and B, are separately patentable. Fifth, patents do not expire, as in Romer (1990).[3] As commented in the

[1] A closest study to ours is Cozzi and Galli (2014) who develop a two-stage R&D model, interpreting each stage as basic and applied research. Crucially, however, they do not consider strategic interactions between firms as they assume "many" firms through free entry.

[2] Scotchmer and Green (1990) assume identical hazard rates for two innovations.

[3] This is also implicit in Scotchmer and Green (1990).

Introduction, these assumptions ensure that no other factors but the general equilibrium effect can reverse the Scotchmer-Green result. Finally, we assume that discoveries of A and B in industry j require flow R&D costs $c_A(j)$ and $c_B(j)$, respectively, in terms of labor units and that these R&D cost differ across industries. This last assumption is the key to our analysis, as asymmetric R&D costs induce industries to respond differently to patent law changes and impact allocation of labor between the R&D and the production sector.

Let Π denote the total "prize" of two-stage R&D competition, i.e., the discounted sum of all future flow profits arising from the production of a new variety. Π is an exogenous parameter in the basic model of Section 3, but will be endogenized in the general equilibrium model of Section 4. Given separate patentability, it is possible that A and B are owned by different R&D firms. In such cases, it is assumed that the owner of A receives the share σ of Π while the remaining share $1-\sigma$ goes to the owner of B. We treat $\sigma \in (0,1)$ as a parameter common to all blueprints in all industries.[4]

The "leader" is the firm that discovers A first; the other firm is the "follower." The main focus of our analysis is on whether or not the leader patents A. In either patent system, the leader secures the profit $\sigma\Pi$ by patenting A. By contrast, the consequence of not patenting A depends on the patent system in effect. In first-to-file, the leader runs the risk of losing the exclusive right to A if the follower discovers and patent A later. In first-to-invent, this risk is minimal because the leader can always establish priority of invention. This difference gives rise to different incentives to patent A under the two patent-issuing rules.

2.1 First-to-File

We begin with the first-to-file rule. Fix an industry $j \in [0,1]$. Suppose that at time t one firm (the leader) discovers A. Figure 1 shows the strategic moves and timing of the subgame played out following the discovery of A. At node D_L the leader chooses between P (= patenting A) and N (= not patenting A). Let $V_F^P(j)$ and $V_F^N(j)$ denote the payoff to the leader, respectively, when she chooses P and when she chooses N (the subscript F indicates the first-to-file rule is under consideration). If she chooses P, the follower skips stage 1 and play moves to node 1, where two

[4] Asymmetry can be introduced into our analysis by distinguishing and ranking industries in terms of the profit-sharing parameter σ_j instead of the R&D costs. The results are similar, so this alternative approach is not pursued here.

firms compete in stage 2. Thus, $V_F^P(j)$ satisfies the following asset-value equation:[5]

$$\rho V_F^P = \beta(\Pi - V_F^P) + \beta(\sigma\Pi - V_F^P) - c_B(j). \tag{1}$$

The right-hand side has the following interpretation. The leader spends flow R&D cost $c_B(j)$ per unit of time and wins the second-stage race with probability β, causing her asset value to change from V_F^P to Π as shown in the first term.[6] At the same time, however, the leader could lose the race with probability β, in which case her asset value changes as indicated by the second term. Thus, the right-hand side of (1) represents the flow net benefit to the leader of patenting A at node D_L. In equilibrium, that benefit equals ρV_F^P, where ρ is the rate of interest.[7] Collecting terms and rearranging yields

$$V_F^P(j) = \frac{\beta\,(1+\sigma)\,\Pi - c_B\,(j)}{\rho + 2\beta}.$$

Next, suppose that the leader chooses not to patent A at node D_L. Then, play moves to node 2 in Figure 1, where an "asymmetric" R&D competition starts, with the leader in stage 2 and the follower in stage 1. If the leader wins this race, the game is over. If instead the follower wins the race, play moves down to node D_F in Figure 1, where the follower has the option of patenting A. We claim that the follower always patents A because there is no reason for secrecy when the leader already has A.[8]

With the follower patenting A, a stage-two race starts at node 3. This stage-two race differs from the one at node 1 in that the leader does not hold the patent for A. Thus, the leader receives $(1-\sigma)\Pi$ if she wins, and nothing if she loses. By the procedure similar to the one that led to equation (1), we can write the expected profit to the leader at node 3 as

$$V_F^{NP}(j) = \frac{\beta\,(1-\sigma)\,\Pi - c_B\,(j)}{\rho + 2\beta}.$$

Now, returning to node D_L, we can calculate the value of not patenting A, $V_F^N(j)$, to the leader as follows. By not patenting A, the leader initiates the asymmetric race described above at node 2. With probability β the leader wins the race, changing her asset value from $V_F^N(j)$ to Π. On the

[5]A capital gain/loss term is suppressed to simplify exposition. It will be explicitly introduced in Section 4 where the basic model is extended to a general equilibrium framework.

[6]More accurately, probability $\beta\Delta$, where Δ is an infinitesimally short time.

[7]In the general equilibrium model of Section 4, we use ρ to denote consumer's rate of time preference. In fact, (1) is equivalent to the steady state version of the asset equation in general equilibrium.

[8]It is readily shown that at node D_F patenting A is the dominant strategy for the follower.

other hand, with probability α, the leader loses the race, causing her asset value to change from $V_F^N(j)$ to $V_F^{NP}(j)$. Thus, $V_F^N(j)$ satisfies the following asset-value equation:

$$\rho V_F^N(j) = \beta \left(\Pi - V_F^N(j)\right) + \alpha \left(V_F^{NP}(j) - V_F^N(j)\right) - c_B(j).$$

Collecting terms yields

$$V_F^N(j) = \frac{\beta\Pi + \alpha V_F^{NP}(j) - c_B(j)}{\rho + \alpha + \beta}.$$

Now we state the leader's decision rule at node D_L; the leader chooses to patent A if and only if $V_F^P(j) \geq V_F^N(j)$. Substituting and simplifying, we restate this condition in

Lemma 1. *In first-to-file, the leader patents A if and only if*

$$c_B(j) - [(1-\sigma)\beta - \sigma(\rho + 2\alpha)]\Pi \geq 0. \tag{2}$$

We now make three assumptions concerning the condition in Lemma 1.

Assumption 1. $c_B(j)$ *is differentiable* on $(0,1)$ and $c_B'(j) > 0$.[9]

Assumption 2. $(1-\sigma)\beta - \sigma(\rho + 2\alpha) > 0$.

Assumption 3. *(i)* $c_B(0) < [(1-\sigma)\beta - \sigma(\rho + 2\alpha)]\Pi$, *and (ii)* $c_B(1) > [(1-\sigma)\beta - \sigma\rho]\Pi$.

Assumption 1 is without loss of generality and says that industries can be ordered in terms of R&D costs for B. Differentiability simplifies the analysis. Assumptions 2 and 3 ensure that A is patented in some industries but not in others.[10]

Assumptions 1 - 3 imply the following:

Lemma 2. *In first-to-file, there exists J_F given by*

$$c_B(J_F) = [(1-\sigma)\beta - \sigma(\rho + 2\alpha)]\Pi \tag{TH_F^0}$$

such that the leader patents A only in industries $j \in [J_F, 1]$

Several comparative-static results follow immediately from Lemma 2.

[9] We do not make a similar assumption regarding $c_A(j)$. Therefore, the analysis that follows in this section does not depend on how $c_A(j)$ changes in j.

[10] Assumption 3(ii) can be replaced with $c(1) > [(1-\sigma)B - \sigma(\rho + 2A)]\Pi$. But, as we will see, (ii) is required for an interior equilibrium in first-to-invent.

Lemma 3. *The threshold industry J_F increases in Π and β, and decreases in α, ρ and σ.*

Intuitions for the lemma are given below:

1. An increase in the prize of innovation Π discourages patenting (raises J_F). This comes from Assumptions 1 and 2. Intuitively, not patenting A prolongs the second-stage R&D, raising the R&D costs. Thus, the decision not to patent A is justified only if the innovation prize is greater. This means that as the prize increases, the leader has less of an incentive to patent A, raising J_F. This result plays an important role in understanding why the Scotchmer-Green result can be reversed in the general equilibrium model in Section 4.

2. An exogenous increase in hazard rate α encourages patenting (lowers J_F). When α is higher, the follower is more likely to catch up, if the leader does not patent A. This makes patenting more attractive to the leader.

3. An exogenous increase in hazard rate β discourages patenting (raises J_F). With a higher β, the leader is more likely to win an asymmetric race (at node 2 in Figure 1), which makes non-patenting more attractive to the leader.

4. An increase in the interest rate ρ encourages patenting (lowers J_F). As explained earlier, not patenting A prolongs stage 2, costing extra R&D expenditure for the leader. A higher interest rate ρ raises the R&D cost even more, prompting patenting in more industries.

5. An increase in the share σ of Π leads to more patenting (lowers J_F). A higher σ means a lower $(1-\sigma)$. This reduces the expected payoff to the leader from non-patenting, and makes patenting more attractive.

2.2 First-to-Invent

We now turn to first-to-invent. The treatment of first-to-invent requires two modifications in Figure 1. First, the payoff box at node 4 changes as indicated in the figure. Second, node 3 becomes irrelevant because the follower cannot patent A.[11]

We begin, as before, with the leader's patent decision at node D_L. When the leader patents A, the consequence is the same as in first-to-file, namely, the leader's payoff $V_I^P(j)$ is identical to

[11] It is assumed that, when the follower files for a patent, the leader blocks it by establishing priority of invention

$V_F^P(j)$;

$$V_I^P(j) = V_F^P(j).$$

(The subscript I denotes the first-to-invent rule.) If the leader chooses not to patent A, an asymmetric race commences at node 2. The expected profit to the leader, $V_I^N(j)$, satisfies

$$\rho V_I^N(j) = \beta \left(\Pi - V_I^N(j)\right) + \alpha \left(V_I^{NP}(j) - V_I^N(j)\right) - c_B(j).$$

To interpret this equation, note that the leader wins this race with probability β, obtaining the entire prize Π. The leader can lose the race with probability α, turning her asset value from $V_I^N(j)$ to $V_I^{NP}(j)$. The latter denotes the value to the leader of the asymmetric race for B that commences when the follower catches up. Collecting terms yields

$$V_I^N(j) = \frac{\beta\Pi + \alpha V_I^{NP}(j) - c_B(j)}{\rho + \alpha + \beta}.$$

This is the value to the leader of not patenting A at node D_L in first-to-invent. This expression is identical to its counterpart in first-to-file, except that $V_I^{NP}(j)$ replaces $V_F^{NP}(j)$. Both these terms represent the value to the leader of the second-stage race commenced when the follower discovers A, but there is a crucial difference. In first-to-file, the follower owns the patent for A, while in first-to-invent the leader owns it due to priority of invention. Thus, the stage-two race at node D_F in first-to-invent is identical to the one at node D_L in first-to-file, that is,

$$V_I^{NP}(j) = \frac{\beta(1+\sigma)\Pi - c_B(j)}{\rho + 2\beta} = V_F^P(j).$$

It is easy to show that $V_I^{NP}(j) > V_F^{NP}(j)$, meaning that, given Π, the leader has a greater incentive not to patent A in first-to-invent than in first-to-file.

We are now in a position to state the leader's decision rule at node D_L. The leader patents A if and only if $V_I^P(j) \geq V_I^{NP}(j)$. Substituting yields

Lemma 4. *In first-to-invent, the leader patents innovation A if and only if*

$$c_B(j) - \left[\beta(1-\sigma) - \rho\sigma\right]\Pi \geq 0. \quad (3)$$

Assumptions 1 - 3 imply the following.

Lemma 5. *In first-to-invent, there exists J_I satisfying*

$$c_B(J_I) = [(1-\sigma)\beta - \sigma\rho]\,\Pi. \qquad (TH_I^0)$$

such that the leader patents A in $j \in [J_I, 1]$.

Lemma 5 shows that an exogenous change in hazard rate α has no effect on the leader's incentive to patent A (i.e., J_I is independent of the hazard α). In first-to-invent, the leader owns A, whether she patents it or not. Therefore, the follower's success rate α cannot affect the leader's patent decision. This contrasts with the first-to-file case. The other parameters (β, Π, ρ and σ) have qualitatively identical effects on the patent decision as in first-to-file and the intuitions are basically the same as before. For later use, we note the following.

Lemma 6. *Both in first-to-file and in first-to-invent, a fall in* Π *encourages patenting, and increases the proportion of patenting industries (i.e., lowers the threshold J).*[12]

We call this result the *prize effect*. In the general equilibrium model of section 4, we discuss how Π links the R&D sector to the rest of the economy and how the prize effect holds the key to the intuitive understanding of the general equilibrium effect of patent policy.

2.3 Three Effects

The next result follows straightforwardly from Lemmas 1-6.

Proposition 1. *Given* Π*, more industries patent A in first-to-file than in first-to-invent (i.e., $J_F < J_I$).*

This result is illustrated in Figure 2. The graphs (TH_F^0) and (TH_I^0) represent the functions given by the left-hand side of (2) and (3), respectively. Both graphs are upward-sloping because $c_B(J)$ is increasing by Assumption 1. By Lemmas 2 and 5, these functions take the value of 0 at the threshold industries J_F and J_I. A comparison between (2) and (3) puts the graph of (TH_F^0) above that of (TH_I^0) as in Figure 2. Proposition 1 follows immediately.

An intuition can be gained by identifying three effects of (not) patenting. First, not patenting A is risky in first-to-file because the follower can catch up and patent it. Hence, the leader is more likely to patent A in first-to-file. We call it the *risk-elimination effect*. This effect is absent in

[12]The intuition is given following Lemma 3.

first-to-invent. Second, patenting allows the follower to bypass stage 1. Since two firms can discover B faster than one, the leader receives the prize (or her share of it) sooner when she patents A. We call it the *expedition effect*. This effect is present in both patent systems. On the other hand, not patenting A makes the follower invent A, giving the leader a head start in the stage-two race. We call it the *detour effect*.

It should now be clear that the risk-elimination effect explains why the leader is more likely to patent A in first-to-file than in first-to-invent, for a given Π. The expedition effect, on the other hand, explains why the two curves are upward sloping in Figure 2. If a flow R&D cost $c_B(j)$ increases, the leader has a greater incentive to end stage 2 sooner. But stage 2 ends sooner if two firms compete (this is the expedition effect). Thus, the leader has a greater incentive to patent A in industries with higher flow costs $c_B(j)$. This and Assumption 1 imply that the (TH_F^0) and (TH_I^0) are upward-sloping.

3 Partial Equilibrium

3.1 The Production Sectors

Having described the R&D sector and the strategic decisions of R&D firms, we turn to the production sectors. As in Romer (1990), the economy has two production sectors. The intermediate-goods sector consists of a continuum of industries $j \in [0,1]$, and industry j uses labor to produce $n_j(t)$ varieties of intermediate goods at time t. The final good sector turns out $Y(t)$ units of the final good from the whole gamut of existing intermediate goods according to the production function:

$$Y(t) = \left(\int_0^1 \int_0^{n_j(t)} x_{ij}(t)^{\theta} di dj \right)^{\frac{1}{\theta}} \tag{4}$$

where $\theta \in (0,1)$ is a parameter and $x_{ij}(t)$ is the flow of output of an intermediate good produced by firm i in industry j at time t.[13] Note that labor is not used in the production of the final good.

Perfect competition prevails in the final good sector. Profit maximization yields the following demand for each intermediate good:

[13] As is known, it is possible to interpret this production function as the Dixit-Stiglitz love-of-variety utility function, implying that consumers directly consume $x_{ij}(t)$ units of variety i produced in industry j.

$$x_{ij}(t) = \frac{p_{ij}(t)^{-\frac{\theta}{1-\theta}}}{\int_0^1 \int_0^{n_j(t)} p_{i'j'}(t)^{-\frac{\theta}{1-\theta}} di' dj'} \cdot \frac{E(t)}{p_{ij}(t)} \tag{5}$$

where $p_{ij}(t)$ denotes the price of the intermediate good i in industry j and $E(t)$ is consumption expenditure.[14] Facing this demand function, each intermediate good firm produces a differentiated product with labor under the patented technology developed in the R&D sector. If each labor unit produces one unit of intermediate-good output, the standard mark-up rule yields $p_{ij} = p = 1/\theta$.[15] Substituting this price simplifies the above demand function to:

$$x_{ij}(t) \equiv x(t) = \frac{E(t)}{N(t)\, p(t)} \qquad \forall\, i, j, \tag{6}$$

where

$$N(t) \equiv \int_0^1 n_j(t)\, dj \tag{7}$$

is the total number of varieties available at time t. Substituting these results into (4), we can write

$$Y(t) = x(t)\, N(t)^{\frac{1-\theta}{\theta}}$$

Given x, the growth rate is given by

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{1-\theta}{\theta} g(t) \quad \text{where } g(t) = \frac{\dot{N}(t)}{N(t)}. \tag{8}$$

This shows that economic growth is driven by increasing varieties.

To link the R&D sector to the growth process, we assume the following. (i) In each sector j, the completion of one round of two-stage R&D is immediately followed by a new round of R&D competition with two new R&D firms. This sequence continues infinitely in time.[16] (ii) The completion of each round of R&D competition in industry j generates δN new varieties in that industry. This assumption ($0 < \delta < 1$) captures positive externality in R&D in a simple and convenient way in the present framework. We show shortly that, with identical Poisson rates α and β in all industries,

[14] A more precise definition of $E(t)$ will be given in Section 4.

[15] Here, the wage is set equal to one in anticipation of the general-equilibrium model of Section 4, in which it serves as the numeraire.

[16] We can introduce an entry cost to be incurred at the beginning of a two-stage patent race. It is assumed to be zero, because the entry cost does not affect our key results.

growth depends crucially on the number of industries in which A is patented.

Before proceeding further, we substitute (6) to express the flow profit for each intermediate good as:

$$\pi(t) = (1-\theta)\frac{E(t)}{N(t)}. \tag{9}$$

Since one round of a two-stage patent race generates $\delta N(t)$ number of new varieties, a total flow profit in an intermediate-goods industry j equals $\delta N(t)\pi(t)$. With permanent patents, the total prize of a patent race is equivalent to the discounted sum of $\delta N(t)\pi(t)$ over an infinite time horizon. Let this sum be denoted by Π. In the remainder of this section we take Π as given. In Section 4 we determine its value endogenously in general equilibrium.[17]

3.2 Steady State

The analysis of this subsection applies equally to first-to-file and first-to-invent. Since innovations occur stochastically, a given industry j is randomly "located" at one of the numbered nodes in Figure 1. For example, in industry $j \in [J, 1]$, $J = J_F, J_I$, where the leader always patents A, firms are either in stage 1 or 2. Thus, the patenting industry randomly changes its location between nodes 0 and 1.

Let AA and BB denote the state in which both firms are in stage 1 and stage 2, respectively, and let Z_{AA}^P and Z_{BB}^P be the number of patenting industries in states AA and BB. Since there are $1-J$ patenting industries in the economy, we have[18]

$$Z_{AA}^P + Z_{BB}^P = 1 - J, \qquad J = J_F, J_I. \tag{10}$$

Industries in state AA discover A with the combined hazard rate of 2α and move to state BB. That is, $2\alpha Z_{AA}^P$ industries move from state AA to state BB, as described in Figure 3(a). Similarly, $2\beta Z_{BB}^P$ industries move from BB to AA, as one round of R&D is completed and a fresh round starts. In steady state, Z_{AA}^P and Z_{BB}^P must be constant, i.e.,

$$2\beta Z_{BB}^P = 2\alpha Z_{AA}^P.$$

[17] Since $\delta N(t)\pi(t) = (1-\theta)E(t)$, fixing Π is equivalent to fixing consumption expenditure $E(t)$.

[18] The subscripts F and I are dropped from Z's unless ambiguity arises.

This and (10) can be used to calculate the steady-state values of industry shares:

$$\frac{Z_{AA}^{P}}{1-J}=\frac{\beta}{\alpha+\beta},\qquad \frac{Z_{BB}^{P}}{1-J}=\frac{\alpha}{\alpha+\beta},\qquad J=J_{F},J_{I}. \tag{11}$$

Turn next to the non-patenting industries $j \in [0, J)$, $J = J_F, J_I$. Such industries can be in three states: AA, BB, and AB, the last of which indicates the asymmetric race in which the leader is in stage 2 and the follower is in stage 1. Thus, non-patenting industries randomly change their locations among three nodes: 0, 2 and 3 in first-to-file and 0, 2 and 4 in first-to-invent. Let Z_{AA}^{N}, Z_{BB}^{N} and Z_{AB}^{N} denote the number of industries in each state. To calculate these values, note that there are J non-patenting industries in the economy, and hence

$$J=Z_{AA}^{N}+Z_{BB}^{N}+Z_{AB}^{N},\qquad J=J_{F},J_{I}. \tag{12}$$

Further, industries in AB move to BB with rate α and to AA with rate β, as shown in Figure 3(b). The number of industries in each state is constant in steady state, implying

$$\begin{aligned}&2\alpha Z_{AA}^{N}=2\beta Z_{BB}^{N}+\beta Z_{AB}^{N},\\&2\beta Z_{BB}^{N}=\alpha Z_{AB}^{N}.\end{aligned}$$

The above three equations yield

$$\frac{Z_{AA}^{N}}{J}=\frac{(\alpha+\beta)\,\beta}{\alpha\beta+(\alpha+\beta)^{2}},\qquad \frac{Z_{BB}^{N}}{J}=\frac{\alpha^{2}}{\alpha\beta+(\alpha+\beta)^{2}},\qquad \frac{Z_{AB}^{N}}{J}=\frac{2\alpha\beta}{\alpha\beta+(\alpha+\beta)^{2}}, \tag{13}$$

where $J = J_F, J_I$. Note that

$$\left.\begin{aligned}z_{k}^{P}&\equiv\frac{Z_{k}^{P}}{1-J},\quad k=AA,BB\\ z_{k}^{N}&\equiv\frac{Z_{k}^{N}}{J},\quad k=AA,BB,AB\end{aligned}\right\}\ J=J_{F},J_{I} \tag{14}$$

can be interpreted as the conditional probability that a given industry is in one of the possible states. This interpretation is valid, irrespective of the patent-issuing rule in effect. For example, (11) implies that a patenting industry is in state AA with probability $z_{AA}^{P}=\beta/\left(\alpha+\beta\right)$.

3.3 Rate of Technical Progress

Recall that each round of R&D competition in industry j creates δN varieties of intermediate goods in that industry. This occurs every time one round of R&D competition is completed and an industry returns to state AA. In a patenting industry, this occurs with Poisson rate 2β as shown in Figure 3(a). In a non-patenting industry, state AA is reached either from state BB or from state AB as in Figure 3(b). The former case occurs with Poisson rate 2β, and the latter with Poisson rate β. These observations imply that the total number of intermediate products $N(t)$, defined in (7), increases according to

$$\dot{N}(t) = \delta N \left(2\beta Z_{BB}^{N} + \beta Z_{AB}^{N}\right) + \delta N 2\beta Z_{BB}^{P}. \tag{15}$$

Then, (11), (13) and (15) can be used to calculate the rate of technical progress

$$\frac{\dot{N}(t)}{N(t)} \equiv g = 2\delta \frac{\alpha\beta}{\alpha+\beta}\left(1 - \frac{\alpha\beta}{\alpha\beta + (\alpha+\beta)^2} J\right), \quad J = J_F, J_I. \tag{TP}$$

This equation shows that an increase in J slows economic growth. This has an intuitive explanation. A higher J implies more non-patenting industries. In non-patenting industries, inventing new blueprints takes a longer time on average because the follower has to "reinvent the wheel" (i.e., innovate A) in more industries. Thus, as J increases, the R&D sector invents fewer new varieties per unit of time, and consequently the economy grows more slowly.

3.4 Equilibrium

In the present partial-equilibrium model, endogenous variables are $J \in \{J_F, J_I\}$ and g. In the first-to-file system, (J_F, g_F) are determined by (TH_F^0) and (TP). In the first-to-invent system, (J_I, g_I) are defined by (TH_I^0) and (TP). In either case, the model can be solved recursively. (TH_F^0) and (TH_I^0) first determine J_F and J_I, respectively and then (TP) maps them to g_F and g_I. This, by Proposition 1, leads to

Proposition 2. *For given* Π, $g_F > g_I$*; growth rate* g *is higher in first-to-file than in first-to-invent.*

The proposition confirms that the Scotchmer-Green result can be extended into a growth context in our basic model, where Π is exogenous. The reason for the result lies in the risk-elimination effect of patenting (identified in Section 2.3), which is present in first-to-file but not in first-to-invent. The next section will demonstrate that the prize effect in Lemma 6, which emerges in general equilibrium settings, works against the risk-elimination effect, so that the result in Proposition 2 can be reversed.

4 General Equilibrium

In this section, we extend the preceding partial-equilibrium model to general-equilibrium settings. We do so by introducing consumers' utility maximization and the labor market, which endogenizes the prize of a patent race Π. We focus on steady state as before.

4.1 Consumers and Profits

We begin with consumers' decision-making. Assume that there are L identical consumers. Each consumer supplies a single unit of labor inelastically and has preferences over the final good represented by the utility function

$$U = \int_0^\infty e^{-\rho t} \ln C(t) dt.$$

Note that ρ now denotes the representative consumer's rate of time preference. In equilibrium, demand $C(t)$ for the final good equals its supply $Y(t)$. The intertemporal optimization results in the Euler condition:

$$\frac{\dot{E}(t)}{E(t)} = r(t) - \rho \tag{16}$$

where $E(t)$ is the total expenditure and $r(t)$ is the instantaneous rate of interest at time t.

In Section 3, we showed that profit per variety $\pi(t)$ is given by (9), and that a total flow profit $\delta N(t) \pi(t)$ is generated from each round of two-stage R&D competition. We now define the prize of a patent race Π as the discounted sum of such flow profits over an infinite time horizon;

$$\Pi(t) = \int_t^\infty e^{-\int_t^\tau r(s)ds} \delta N(\tau) \pi(\tau) d\tau = \frac{\delta(1-\theta)}{\rho} E(t). \tag{17}$$

The second equality in (17) makes use of (16) with $\dot{E}(t) = 0$. Using (17), we can rewrite the threshold conditions (TH_F^0) and (TH_I^0) as

$$c_B(J_F) = [(1-\sigma)\beta - \sigma(\rho + 2\alpha)] \frac{\delta(1-\theta)}{\rho} E, \tag{TH_F}$$

$$c_B(J_I) = [(1-\sigma)\beta - \sigma\rho] \frac{\delta(1-\theta)}{\rho} E. \tag{TH_I}$$

4.2 The Values of R&D Firms

In describing the R&D sector in Section 2, we deliberately ignored the term capturing capital gains/losses in the asset equations of R&D firms. It was an innocuous shortcut to ease the exposition of the model. In this section, we explicitly take into account the fact that the value of R&D firms changes as the economy grows.

First, let us consider the leader in a patenting industry j under the first-to-file system. Its value, defined in (1) in the partial equilibrium model, is now given by

$$r(t)\,V_F^P(t) = -c_B(j) + B(\Pi(t) - V_F^P(t)) + B(\sigma\Pi(t) - V_F^P(t)) + \dot{V}_F^P(t) \tag{18}$$

where the last term represents a capital gain/loss. Note that we have $\dot{E}_t = \dot{V}_F^P = 0$ in steady state, given that wage is the numeraire. Using (16), therefore, (18) is reduced to (1). By the same logic, one can show that the value functions used in the partial equilibrium model are all valid for this extended model. Using (17), we can now rewrite the threshold conditions (TH_F^0) and (TH_I^0) as

$$c_B(J_F) = [(1-\sigma)\beta - \sigma(\rho + 2\alpha)]\frac{\delta(1-\theta)}{\rho}E, \tag{TH_F}$$

$$c_B(J_I) = [(1-\sigma)\beta - \sigma\rho]\frac{\delta(1-\theta)}{\rho}E. \tag{TH_I}$$

4.3 Labor Market

To close the model, we introduce the labor market. In equilibrium, the total supply of labor L is allocated between the R&D sector (L^R) and the intermediate goods sector (L^M). Since each unit of labor produces one unit of an intermediate good, the total demand for labor in the intermediate-goods sector is given by

$$L^M(t) = \int_0^1 \int_0^{n(j)} x_{ij}(t)\,di dj = \theta E(t),$$

where the second equality follows from (5). Naturally, an increase in E raises labor demand in manufacture.

To calculate the demand for labor in the R&D sector, recall that $c_A(j)$ and $c_B(j)$ denote the number of workers employed in stage 1 and stage 2 in industry j, respectively. Let us first consider a patenting industry $j \in [J, 1]$, $J = J_F, J_I$. We showed already that, in steady state, z_{AA}^P and z_{BB}^P are the probability that industry j is in states AA and BB, respectively. Therefore, the total number of R&D workers in industry j is $2c_A(j)\, z_{AA}^P + 2c_B(j)\, z_{BB}^P$. Making use of this, we can compute the number of R&D workers in the patenting industries:

$$\int_J^1 \left(2c_A(j)\, z_{AA}^P + 2c_B(j)\, z_{BB}^P\right) dj. \tag{19}$$

An analogous procedure shows that there are $2c_A(j)\, z_{AA}^N + 2c_B(j)\, z_{BB}^N + (c_A(j) + c_B(j))\, z_{AB}^N$ R&D workers in a non-patenting industry $j \in [0, J)$, $J = J_F, J_I$. The first two terms arise from symmetric states AA and BB, while the third represents the labor demand in asymmetric state AB. Thus, the total amount of R&D labor employed in non-patenting industries equals

$$\int_0^J \left[2c_A(j)\, z_{AA}^N + 2c_B(j)\, z_{BB}^N + (c_A(j) + c_B(j))\, z_{AB}^N\right] dj. \tag{20}$$

Combining (19) and (20), we can show, after much algebra, that the total number of workers in the R&D sector, L^R, equals

$$L^R(t) = \bar{\chi} + \chi(J), \qquad J = J_F, J_I$$

where

$$\begin{aligned} \bar{\chi} &\equiv \frac{2}{\alpha+\beta}\left(\beta \int_0^1 c_A(j)\, dj + \alpha \int_0^1 c_B(j)\, dj\right) > 0 \\ \chi(J) &\equiv \frac{2\alpha^2\beta}{\left(\alpha\beta + (\alpha+\beta)^2\right)(\alpha+\beta)} \int_0^J \left[c_A(j) - c_B(j)\right] dj. \end{aligned}$$

Note that $\bar{\chi}$ is independent of J but that $\chi(J)$ depends on J, leading to

Lemma 7. *In general equilibrium, an increase in* J $(= J_F, J_I)$ decreases demand for R&D labor if and only if $c_A(j) < c_B(j)$.

To understand Lemma 7 intuitively, consider the following ratios constructed from (11) and (13):

$$\begin{aligned} \frac{z_{BB}^P}{z_{AA}^P} &= \frac{\alpha}{\beta} \\ \frac{z_{BB}^N + z_{AB}^N/2}{z_{AA}^N + z_{AB}^N/2} &= \frac{\alpha}{\beta} \cdot \frac{\alpha+\beta}{2\alpha+\beta} \end{aligned}$$

These ratios indicate the relative probabilities that a firm is in stage 2 of R&D competition (relative to stage 1) in a patenting and a non-patenting industry, respectively. Computation shows that the former ratio exceeds the latter, meaning that a patenting industry is more likely to be in stage 2 than a non-patenting industry. Thus, if $c_A(j) < c_B(j)$, a patenting industry employs more labor for R&D on average than a non-patenting industry, and hence an increase in J (which increases the number of non-patenting industries) reduces demand for R&D labor.

The labor market clears if $L = L^M + L^R$, that is,

$$L = \theta E + \bar{\chi} + \chi(J), \qquad J = J_F, J_I. \tag{LM}$$

(LM) shows that the relationship between consumer expenditure and the demand for R&D labor depends on how $\chi(J)$ changes in J (see Lemma 7).

4.4 Steady State Equilibrium

The following result defines a steady state equilibrium of the general equilibrium model.

Proposition 3. *In steady state, the equilibrium growth rate in the first-to-file system is determined by (TH_F), (TP) and (LM), and it is derived from (TH_I), (TP) and (LM) for the first-to-invent rule.*

These are the key functions in our analysis. First, (17) maps E positively to Π. Then, (TH_F) and (TH_I) relate Π positively to J by Lemma 5. Finally, (TP) relates J $(= J_F, J_I)$ negatively to g. The reason for the negative relationship between J $(= J_F, J_I)$ and g is familiar by now; non-patenting industries take longer times to complete each round of R&D, and hence an increase in the share of non-patenting industries in the economy (an increase in J) slows economic growth. Therefore, these functions together imply that an increase in E is negatively related to economic growth.[19]

Since labor market-clearing requires that E and J satisfy condition (LM), the (LM) function and the (TH_k) functions $(k = F, I)$ jointly determine the equilibrium J $(= J_F,\ J_I)$ and E. The (TH_k) functions defines a positive relationship between E and J, while the relationship defined by the (LM) function can be positive or negative. As we will see shortly, how the (LM) function relates E to J holds the key to understanding the nature of steady state equilibrium.

[19] (17) shows that Π (hence profit) is a positively linear function of expenditure E, meaning that profit and growth are negatively related. This is consistent with the result of Aghion, et al. (2005) that there is an inverted U relationship between competition and growth, implying that higher profit can discourage growth.

4.4.1 Unique Interior Equilibrium

In this subsection we consider three cases in which there is a unique interior equilibrium. Multiple interior equilibria are discussed in the following subsection.

Lemma 7 implies that interindustry R&D cost differences are crucial in understanding the nature of steady state equilibrium. Let us begin with the case of $c_A(j) = c_B(j)$ for all $j \in [0,1]$. This special case is depicted in Figure 4. The discussion above implies that the (LM) function is independent of J, so it is represented by the vertical line in Figure 4. Therefore, the (LM) equation alone determines the equilibrium E uniquely. From the preceding discussion, the (TH_F) and (TH_I) curves are both upward sloping as shown in the figure. The intersections of these functions with the (LM) curve determine the equilibrium J_I and J_F. Now, Proposition 1 states that $J_I > J_F$ for a given Π and hence E, implying that the (TH_F) curve lies above the (TH_I) curve, as in Figure 4. Thus, $J_I > J_F$ implies that $g_I < g_F$ in steady state equilibrium. The vertical (LM) function in Figure 4 implies that patent law changes generate no general equilibrium effect. Therefore, the present case is essentially equivalent to the partial equilibrium model of Section 3. Note that this result is driven solely by the risk-elimination effect of patenting, identified in Section 2.3. The prize effect identified in Lemma 6 is absent because E (hence Π) is the same for both patenting systems. A reversal of the Scotchmer-Green result is possible only when the risk-elimination effect is outweighed by the prize effect, which emerges only for $c_A(j) \neq c_B(j)$.

Turning to general cases of unequal R&D costs across industries, we distinguish the following three cases:

Case (i) $c_A(j) \geq c_B(j)$, i.e. $\chi'(J) \geq 0$ for $j \in [0,1]$ and

$$\frac{c_B(0)}{\Delta_F} < \frac{L - \bar{\chi} - \chi(0)}{\theta}, \qquad \frac{c_B(1)}{\Delta_I} > \frac{L - \bar{\chi} - \chi(1)}{\theta} \tag{21}$$

Case (ii) $c_A(j) < c_B(j)$, i.e. $\chi'(J) < 0$ for $j \in [0,1]$ and (21)

Case (iii) $c_A(j) < c_B(j)$, i.e. $\chi'(J) < 0$ for $j \in [0,1]$ and

$$\frac{c_B(0)}{\Delta_F} > \frac{L - \bar{\chi} - \chi(0)}{\theta}, \qquad \frac{c_B(1)}{\Delta_I} < \frac{L - \bar{\chi} - \chi(1)}{\theta} \tag{22}$$

where

$$\Delta_F \equiv [(1-\sigma)\beta - \sigma(\rho + 2\alpha)] \frac{\delta(1-\theta)}{\rho}, \qquad \Delta_I \equiv [(1-\sigma)\beta - \sigma\rho] \frac{\delta(1-\theta)}{\rho}.$$

By Lemma 7, the (LM) curve is downward-sloping (or vertical) in (E, J) space in Case (i) and upward-sloping in Cases (ii) and (iii).

Let us consider each case in turn. Figure 5 illustrates Case (i), where $c_A(j) > c_B(j)$. [20] (The special case, with $c_A(j) = c_B(j)$, was already examined above.) By Lemma 7, the (LM) curve is downward-sloping.[21] Figure 5 indicates that $J_F < J_I$. Then, the growth equation (TP) implies $g_F > g_I$; first-to-file promotes faster economic growth.[22] This is qualitatively the same result as in the partial equilibrium case. To develop the intuition which will turn out useful later, note that the E, and hence Π, is higher in first-to-file than in first-to-invent, which by Lemma 6 implies that the prize effect makes patenting less attractive in first-to-file than in first-to-invent. However, in the present case, the prize effect is dominated by the risk-elimination effect that works against patenting in determining J_F and J_I, as shown in Figure 5.

We turn to Cases (ii) and (iii), where $c_A(j) < c_B(j)$. In Lemma 7, we showed that for $c_A(j) < c_B(j)$ the (LM) curve slopes upward, implying a positive relationship between E and J. This relation arises because an increase in J reduces labor demand in R&D, thereby shifting labor to the production sector and raising E and hence prize Π. In general, as $c_B(j)$ increases relative to $c_A(j)$, a rise in J brings about a larger increase in E, flattening the (LM) curve. As we will see shortly, this result can amplify the prize effect, possibly reversing the Scotchmer-Green result.

Let us first consider Case (ii). In this case, the difference between $c_B(j)$ and $c_A(j)$ is relatively small, so that (LM) cuts the the threshold conditions (TH_F) and (TH_I) "from below," as shown in Figure 6. As a consequence, $J_F < J_I$, and hence $g_F > g_I$, which is qualitatively equivalent to Case (i).[23] Intuitively, Figure 6 shows that E (hence Π) is lower in first-to-file than in first-to-invent, implying that patenting is a more attractive option in first-to-file. In this sense, the prize effect reinforces the risk-elimination effect of patenting so that g_F is even greater relative to g_I compared with Case (i). However, this result dramatically changes in Case (iii), to which we turn next.

[20] In Figure 5, each side of the two inequality conditions in (21) defines the "corner values" of the (TH_F), (TH_I) and (LM) curves at $J = 0$ and $J = 1$.

[21] This curve starts from a point to the right of the (TH_F) curve on the horizontal axis and ends at a point to the left of the (TH_I) curve for $J = 1$.

[22] (21) guarantees the existence of a unique interior equilibrium.

[23] The inequality conditions in (21) ensure the existence of an interior equilibrium.

Figure 7 illustrates the case, where the (LM) curve intersects the (TH_k) curves $(k = F, I)$ "from above" unlike in Cases (i)-(ii). This arises if the difference between $c_B(j)$ and $c_A(j)$ is sufficiently large. It is clear from the figure that $J_F > J_I$ and hence $g_F < g_I$; first-to-invent promotes faster economic growth than first-to-file, a result that contrasts sharply with the Scotchmer-Green result and those from the previous cases. This reversal can be explained intuitively as follows. In the present case, E (hence Π) is smaller in first-to-invent than in first-to-file in equilibrium. That is, the prize effect, which is endogenously determined through the labor market, is so large that it dominates the risk-elimination effect, inducing A to be patented in a greater number of industries in first-to-invent, thereby leading to a higher growth rate comapred with first-to-file.

Unlike Cases (i) and (ii), Case (iii) can have, in addition to the interior equilibrium, the corner solutions at

$$\begin{cases} E_0 = \dfrac{L - \bar{\chi} - \chi(0)}{\theta} \\ J_F = J_I = 0 \end{cases} \qquad \begin{cases} E_1 = \dfrac{L - \bar{\chi} - \chi(1)}{\theta} \\ J_F = J_I = 1 \end{cases}$$

where E_0 and E_1 are consumption expenditure when $J = 0$ and $J = 1$, respectively, for $J = J_F, J_I$. Figure 8 is instrumental in explaining this result. To ease exposition, only the (TH_I) line (for the first-to-invent system) is drawn for three different values of $\Pi = \dfrac{\delta(1-\theta)}{\rho} E$ in (17). The middle curve defines the threshold industry J_I, which corresponds to the interior equilibrium in Figure 7. The upper line, on the other hand, characterizes a corner solution C in Figure 8, which corresponds to $E_0 = \dfrac{L - \bar{\chi} - \chi(0)}{\theta}$ in Figure 7. For the existence of this equilibrium, the following condition must hold:

$$\frac{c_B(0)}{\Delta_F} \geq E_0 = \frac{L - \bar{\chi} - \chi(0)}{\theta}. \tag{23}$$

The left inequality comes from the fact that (2) is positive at C, and the right equality is due to the labour market condition (LM). Similarly, a corner equilibrium D in Figure 8 corresponds to $E_1 = \dfrac{L - \bar{\chi} - \chi(1)}{\theta}$ in Figure 7, and requires

$$\frac{c_B(1)}{\Delta_F} \leq E_1 = \frac{L - \bar{\chi} - \chi(1)}{\theta}. \tag{24}$$

The same reasoning applies to the first-to-file case, yielding similar conditions as above. Importantly, the conditions (23) and (24) and similar conditions for the first-to-file system are all consistent with (22). That is, a unique interior equilibrium coexists with those corner solutions.

The next proposition summarizes what we found so far in this section:

Proposition 4. *There exists an equilibrium such that* $J_F > J_I$ *and* $g_F < g_I$ *for* $c_A(j) < c_B(j)$.

4.4.2 Multiple Interior Equilibria

In the preceding section, we focused on cases of unique interior equilibrium. In Case (i), there is no other equilibria since the (LM) curve is negatively sloped and hence can cross the (TH_k) curves, $k = F, I$ only once. However, in Cases (ii) and (iii), there can be multiple equilibria because the (LM) and (TH_k) curves, $k = F, I$, are both positively sloped. In this subsection we examine such possibilities.[24]

In general, Case (ii) admits odd numbers of interior equilibria. Figure 9 shows three equilibria in each patent system. Let us label the three pairs of equilibrium borderline industries by $\left(J_F^1, J_I^1\right)$, $\left(J_F^2, J_I^2\right)$ and $\left(J_F^3, J_I^3\right)$. Note that at $\left(J_F^2, J_I^2\right)$ and $\left(J_F^3, J_I^3\right)$, the (LM) curve cuts the (TH_k) curves $(k = F, I)$ "from below", whereas at $\left(J_F^1, J_I^1\right)$ the (LM) curve cuts the (TH_k) curves $(k = F, I)$ "from above". In the latter case, we have $J_F^1 > J_I^1$ and hence $g_F^1 < g_I^1$, which reverses the Scotchmer-Green result. The same reasoning can be applied to the case of m equilibria $(m = 3, 5, 7, 9...)$.

In Case (iii), by contrast, interior equilibria can occur in odd or even numbers. Figure 10 depicts the case of three equilibria, with the Scotchmer-Green result reversed at $\left(J_F^2, J_I^2\right)$ and $\left(J_F^3, J_I^3\right)$ where the (LM) curve cuts the (TH_k) curves $(k = F, I)$ "from above". When there are even numbers of interior equilibria, there can be two types. The first type arises if the following conditions hold:

$$\frac{c_B(0)}{\Delta_F} < \frac{L - \bar{\chi} - \chi(0)}{\theta}, \qquad \frac{c_B(1)}{\Delta_F} < \frac{L - \bar{\chi} - \chi(1)}{\theta}. \tag{25}$$

Figure 11 shows the case of two interior equilibria, where the above conditions are met. It is clear from the figure that the the Scotchmer-Green result is reversed at $\left(J_F^1, J_I^1\right)$, where the (LM) curve cuts the (TH_k) curves $(k = F, I)$ "from above". We have the second type if the following inequalities hold:

$$\frac{c_B(0)}{\Delta_I} > \frac{L - \bar{\chi} - \chi(0)}{\theta}, \qquad \frac{c_B(1)}{\Delta_I} > \frac{L - \bar{\chi} - \chi(1)}{\theta}. \tag{26}$$

Figure 12 illustrates the case with two interior equilibria, where the above inequalities are satisfied. Again, the Scotchmer-Green result is reversed at $\left(J_F^2, J_I^2\right)$, where the (LM) curve cuts the (TH_k)

[24] We focus on $\chi'(J) > 0$ and do not consider the case where the sign of $\chi'(J)$ changes, i.e. $\chi'(J) \gtreqless 0$, because the latter sheds little new light on the issue at hand, though it is more general.

curves ($k = F, I$) "from above." The case of m equilibria where $m = 2, 3, 4, 6, ...$ can be analyzed similarly.

We have established the following result:

Proposition 5. *If there is a multiplicity of interior equilibria, there is at least one interior equilibrium in which the Scotchmer-Green result is reversed .*

4.5 Discussion: How "Plausible" Is It?

We have seen that in cases of multiple equilibria the the Scotchmer-Green result is reversed at least in one interior equilibrium. But how "plausible" is it? We now address this question. Since reversals occur in the presence of multiple equilibria, one possible way to reduce the number of equilibria is to appeal to stability analysis. However, it is not particularly useful in the present analysis since we focus on steady state throughout. Instead, we examine whether our model is consistent with the standard growth models and empirical works in terms of the predictions they make. For this purpose, we focus on the rate of subjective time preference ρ. There are two reasons for the selection of this criterion. Firstly, the effect of ρ is widely known in growth models. Secondly, reversals occur only in the general equilibrium model, where consumer spending plays an important role.

Intuitively, a greater degree of patience is good for growth. Growth is driven by accumulation of physical and human capital and technical progress, all of which require investment in one form or another. And patience is an important determinant of deferred consumption and hence investment, affecting income levels and growth. Indeed, this intuition is consistent with the standard models of economic growth. According to the Ramsey-Cass-Koopmans model, income levels are higher with a lower rate of time preference. In the endogenous growth literature, both the AK model and the Lucas model of human capital accumulation imply faster growth when consumers are more patient. The standard R&D-based models of Romer (1990), Aghion and Howitt (1992) and Grossman and Helpman (1991) on which the current model is based, as well as a myriad of other growth models yield similar predictions.

Turning to empirical evidence, Dohmen, et. al. (2016) directly tackle the issue of how patience affects income/growth. Using a dataset of 8,000 individuals from 76 countries, they find that patience alone explains about 40% of income variations across samples in a univariate regression. Their study also reports a strong correlation between patience and economic growth in the medium-run growth after World War II as well as over the past 200 years. Hübner and Vannoorenberghe (2015) conduct a similar analysis. Using a panel of 89 countries, they find a sizable impact of patience on income per worker, total factor productivity, and capital stock in 2000. In a different approach, Galor and Özak (2016) argue that growth-conducive environments lead to a greater degree of patience. On an individual level, Mischel et al. (1992) report that patient children are more likely to take formal education and acquire higher incomes in their later life. From a historical perspective, Clark (2007) argue that patience drives economic development. All these studies indicate that growth is higher with a lower value of the rate of time preference ρ.

We now examine whether our model yields a prediction which are consistent with our intuition, the standard growth models and the empirical evidence above. In our model, changes in ρ affect the threshold conditions (TH_F) and (TH_I) only. In particular, a fall in ρ (a greater degree of patience) causes the (TH_k) curve, $k = F, I$, to shift up. With this result on hand, consider first the unique interior equilibrium cases. In Cases (i) and (ii) in Figures 4-6, the upward shifting (TH_F) and (TH_I) curves cause J_F and J_I to rise, implying slower economic growth. This is contrary to the evidence discussed above. In Figures 7, on the other hand, a lower ρ causes J_F and J_I to fall, leading to faster economic growth. This is consistent with the predictions of the standard growth models, the evidence, and our intuition regarding the effect of a greater degree of patience. Thus, an equilibrium is consistent with theory and evidence only where the (LM) curve cuts the (TH_F) and (TH_I) curves "from above", that is, the Scotchmer-Green result is reversed.

It is easily confirmed that the above conclusion holds when there are multiple interior equilibria, as in Figures 9-12. That is, only when the (LM) curve intersects the (TH_F) and (TH_I) curves "from above", the equilibrium is consistent with empirical evidence and the predictions from the standard growth models, and in such equilibriums the Scotchmer-Green result is reversed.

5 Conclusion

In the paper, we present an R&D-based model of economic growth, where discoveries of blueprints for intermediate goods take two innovations, and where innovators strategically decide whether or not to patent their innovations in the spirit of Scotchmer and Green (1990). We assume asymmetric R&D costs across intermediate industries and explore how incentives to patent innovations are determined by the choice between two patent-issuing rules: first-to-file and first-to-invent. In addition to presenting a growth model with these new features, our focus is on the question of which rule is more conducive to product development and economic growth. Our conclusion is that first-to-invent promotes faster growth than first-to-file. Thus, if our analysis is correct, we are in a "wrong" patent regime.

To save space we present the analysis based on inter-industry R&D cost differences. However, our result is robust if industries are differentiated with respect to the distributive shares of the prize between innovators of first-stage and second stage innovations (see footnote 5). Admittedly, other assumptions can potentially influence the conclusion of our model. Thus it is important to point out two obvious limitations of our model.

First, to follow the Scotchmer-Green framework, we assume that the arrival rates of intermediate and final innovations are fixed. This assumption is made to extend their analysis to a growth context and also useful in simplifying the general-equilibrium model. However, it comes at a cost. Suppose an economy switches from first-to-invent to first-to-file. Its total impact can be decomposed into two component effects: (1) the effect on growth realized for given arrival rates, and (2) the remaining effect on growth through changes in arrival rates. The present analysis captures (1) only. A future work should try to endogenize arrival rates,. Such work will also verify whether the result of Miyagiwa and Ohno (2015) mentioned in the Introduction holds in a growth context. Second, there is no free entry into the R&D sector in our model. This may be justifiable due to fixed entry cost. However, allowing free entry is a natural extension of the model. However, with free entry, the underlying mechanism that operates through consumer spending and the labor market and partitions of industries into patenting and non-patenting nonetheless remains intact and hence we conjecture that free entry is unlikely to alter our conclusion. Finally, the robustness of our result should be checked in alternative models of growth, for example, those based on quality improvement. We leave these extensions for future research.

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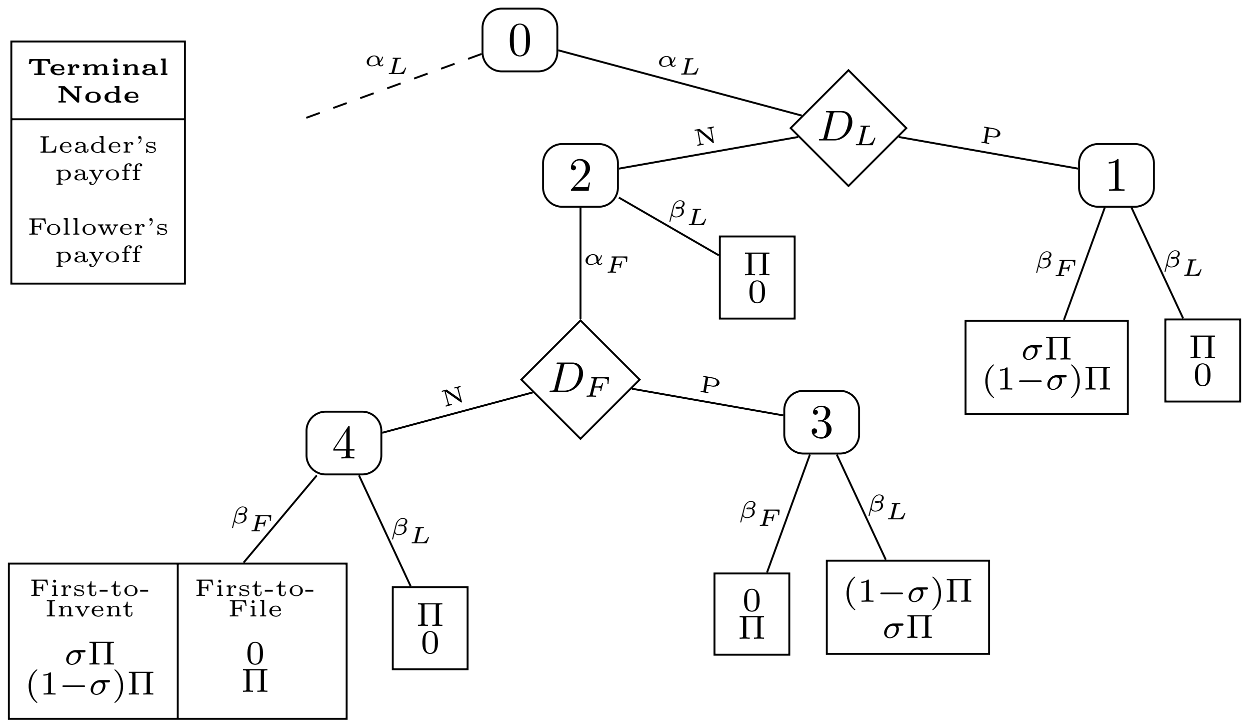

Figure 1: The leader decides on whether or not to patent the intermediate innovation at node D_L, and the follower decides on patenting at node D_F. α_L and β_L mean that the leader succeeds in the intermediate and final innovations, respectively. Similarly, α_F and β_F are for the follower. Firms conduct R&D at nodes 0, 1, 2, 3 and 4.

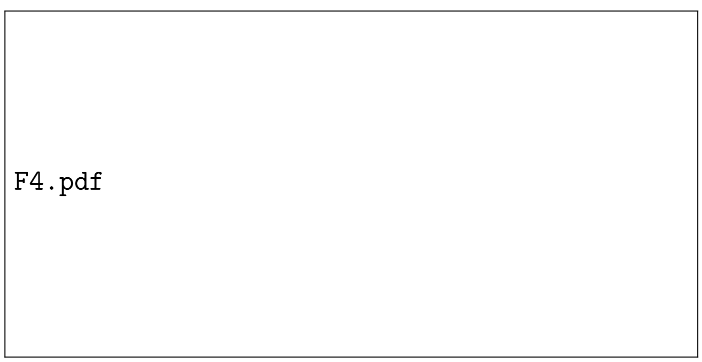

Figure 2: Determination of the threshold industries.

F11.pdf

Figure 3: Transition of industries.

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Figure 4: Partial equilibrium in Section 3 is essentially equivalent to a general equilibrium model with $c_A(j) = c_B(j)$ for all j.

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Figure 5: Equilibrium in Case (i).

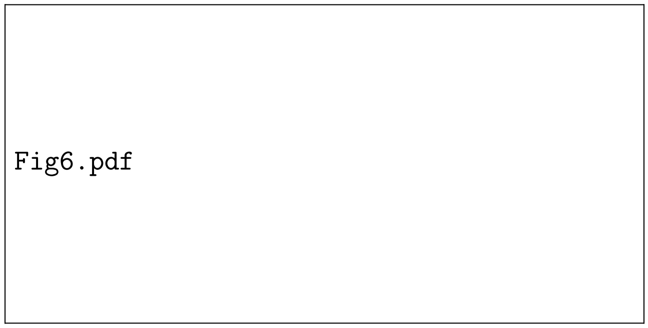

Figure 6: Equilibrium in Case (ii).

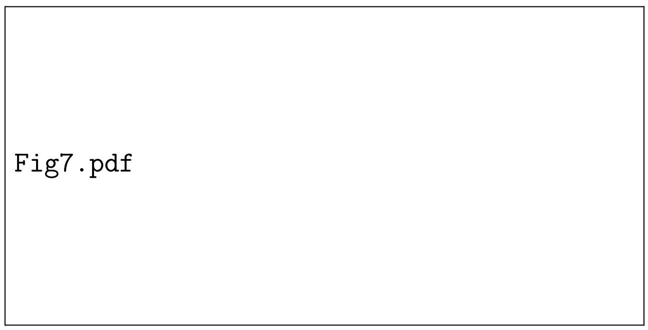

Figure 7: Equilibrium in Case (iii).

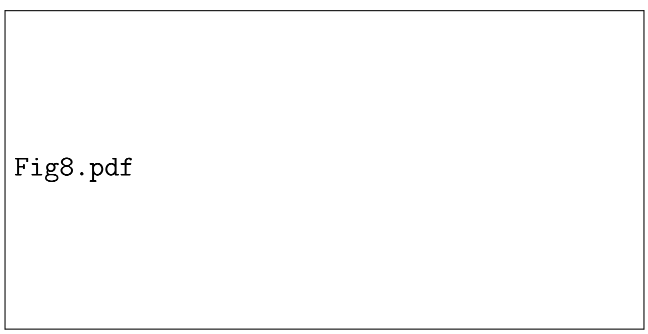

Figure 8: A unique interior equilibrium coexists with two corner solutions in Case (iii).

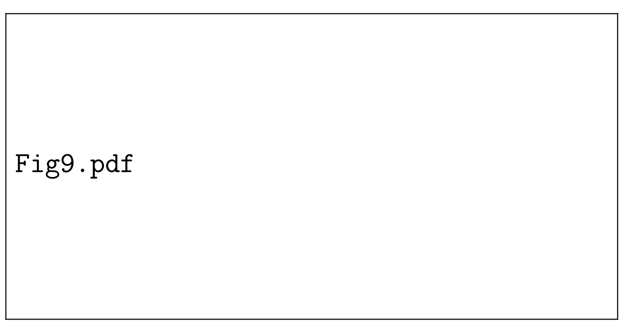

Figure 9: The reversal of the Scotchmer-Green result occurs at $\left(J_I^1, J_F^1\right)$.

Fig10.pdf

Figure 10: The reversal of the Scotchmer-Green result occurs at $\left(J_I^2, J_F^2\right)$ and $\left(J_I^3, J_F^3\right)$.

Fig11.pdf

Figure 11: The reversal of the Scotchmer-Green result occurs at $\left(J_I^1, J_F^1\right)$.

Fig12.pdf

Figure 12: The reversal of the Scotchmer-Green result occurs at $\left(J_I^2, J_F^2\right)$.