Understanding the International Elasticity Puzzle*

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Abstract

International trade studies have higher macro elasticity measures compared to international finance studies, which has evoked mixed policy implications regarding the effects of a change in trade costs versus exchange rates on welfare measures. This so-called *international elasticity puzzle* is investigated in this paper by drawing attention to the alternative strategies that the two literatures use for the aggregation of foreign products in consumer utility functions. Using the implications of having a finite number of foreign countries in nested CES frameworks that are consistent with the two literatures, the discrepancy between the elasticity measures is explained by showing theoretically and confirming empirically that the macro elasticity in international trade is a weighted average of the macro elasticity in international finance and the corresponding elasticity of substitution across products of foreign source countries.

JEL Classification: F12, F14, F41

Key Words: International Elasticity Puzzle; International Trade and Finance.

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Abstract

International trade studies have higher macro elasticity measures compared to international finance studies, which has evoked mixed policy implications regarding the effects of a change in trade costs versus exchange rates on welfare measures. This so-called *international elasticity puzzle* is investigated in this paper by drawing attention to the alternative strategies that the two literatures use for the aggregation of foreign products in consumer utility functions. Using the implications of having a finite number of foreign countries in nested CES frameworks that are consistent with the two literatures, the discrepancy between the elasticity measures is explained by showing theoretically and confirming empirically that the macro elasticity in international trade is a weighted average of the macro elasticity in international finance and the corresponding elasticity of substitution across products of foreign source countries.

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1 Introduction

International trade studies have higher macro elasticity measures compared to international finance studies. Since price movements due to policy changes are converted into welfare adjustments through these elasticities, this observation evokes mixed policy implications regarding the effects of trade costs in international trade versus the effects of exchange rates in international finance (e.g., see Ruhl (2008)). Due to these mixed implications on welfare, this observation is called *the international elasticity puzzle*.

In order to have a better idea about the magnitude of this puzzle, consider a short summary of studies given in Table 1. Although elasticity measures differ across these studies, international finance studies mostly follow Backus, Kehoe, and Kydland (1994) with a macro elasticity value of about 1.5, while international trade studies mostly follow Anderson and Van Wincoop (2004) or recently Simonovska and Waugh (2014a) and Simonovska and Waugh (2014b) with a macro elasticity value of about 5.[1] It is implied that if we directly employ these numbers in a policy analysis, say, in order to investigate the effects of a foreign price change due to tariffs or exchange rates, international trade studies imply quantity changes that are at least three times the international finance studies.

This paper attempts to understand *the international elasticity puzzle* by drawing attention to the alternative strategies the two literatures have for the aggregation of foreign products in consumer utility functions. In particular, while the majority of international finance models include a *unique* foreign country (in their two-country frameworks) in order to have an understanding of the macroeconomic developments in the home country, the majority of international trade models include *multiple* foreign countries in order to investigate the bilateral trade patterns of the home country. Since having alternative numbers of foreign countries is reflected as alternative macro elasticity measures between the two literatures in a nested constant elasticity of substitution (CES) framework, as shown in this paper, *the international elasticity puzzle* can be understood by paying attention to the alternative ways that foreign products are aggregated in the two literatures.

[1] Since the trade elasticity used in new trade models, such as Eaton and Kortum (2002), corresponds to the elasticity of substitution across countries (including home country) minus one in international trade as shown in studies such as by Anderson and Van Wincoop (2004), the commonly used trade elasticity of about 4 suggested by Simonovska and Waugh (2014a) and used by new trade models corresponds to the elasticity of substitution across countries of about 5.

Regarding the details, when a finite number of goods and foreign countries is considered in nested CES frameworks that are consistent with both literatures, this paper finds alternative expressions for the price elasticity of demand as a function of the macro elasticity measures in the two literatures. In order to investigate the conditions under which the two literatures have the very same policy implications (e.g., regarding changes in trade costs versus exchange rates), this paper equalizes the price elasticity measures between the two literatures. This strategy results in an expression that connects the alternative macro elasticity measures in the two literatures, where good-level details are cancelled out during the equalization of the price elasticity measures. In particular, it is theoretically shown that the macro elasticity in international trade is a weighted average of the macro elasticity in international finance and the elasticity of substitution across products of different foreign source countries, where the weight is shown to depend on the number of foreign countries and home expenditure shares. Therefore, the alternative strategies in the two literatures for the aggregation of foreign products are reflected as alternative macro elasticity measures between the two literatures.

The implications of equalizing the price elasticity of demand measures between the two literatures are also tested empirically. Since this investigation requires data on both domestic and foreign trade, it cannot be achieved by using any international trade data set, where domestic trade are not recorded. As an alternative, this paper uses the available trade data within the U.S. by considering interstate trade as foreign trade and intrastate trade as domestic trade. The results based on the estimation of macro elasticity measures in both literatures confirm the theoretical solution provided in this paper that the macro elasticity in international trade is a weighted average of the macro elasticity in international finance and the elasticity of substitution across products of different foreign sources. Therefore, the discrepancy between the macro elasticity measures in the two literatures can in fact be understood by paying attention to the alternative ways that foreign products are aggregated in the two literatures.

Compared to the literature, the nested CES framework in this paper works in a similar way to the one introduced by Atkeson and Burstein (2008) who have shown that considering finite number of goods (and thus good-specific market shares implying variable markups) is essential to explain why export and import prices show substantial and systematic deviations from relative purchasing power parity (PPP) in comparison with source country producer prices. In comparison, having a finite number of source countries in this paper results in having source-country-specific expenditure shares entering into price elasticity measures. This is in contrast to studies such as by Gali and Monacelli (2005) considering infinite number (a continuum) of source countries, where source-country-specific expenditure shares are ineffective in the calculation of price elasticity measures. Since good-level details (and thus the corresponding expenditure shares) are shown to cancelled out during the equalization of the price elasticity measures between the two literatures, having a finite number of source countries (rather than having a finite number of goods) is the key to understand the international elasticity puzzle. Such a theoretical implication (of having sizable source-country-specific expenditure shares) is highly supported by the data as well; e.g., within the U.S., the expenditure share of Kentucky on the products imported from Ohio is about 48% for the second quarter of 2012, while the expenditure share of Delaware on the products imported from New York is about 43% for the first quarter of 2012 (according to the data used in the empirical investigation, below).

This paper deviates from the existing literature due to two main reasons. First, while existing studies in the literature have attempted to understand the puzzle by using the very same functional forms to aggregate across foreign products, this paper draws attention to the difference between the strategies in the aggregation of foreign products between the two literatures. In particular, while providing several supply-side explanations to the puzzle, studies such as by Ruhl (2008), Fitzgerald and Haller (2014), Ramanarayanan (2017), Crucini and Davis (2016) or Feenstra, Luck, Obstfeld, and Russ (2018) all consider the very same functional forms between the two literatures in order to aggregate across foreign products. However, a common aggregation strategy is not consistent with either international trade or international finance studies, where the former aggregates across source countries in the upper-tier (e.g., see Anderson and Van Wincoop (2003), Anderson and Van Wincoop (2004), Head and Ries (2001), Hillberry and Hummels (2013), or Hummels (2001), among many others), and the latter aggregates across home and foreign countries in the upper-tier (e.g., see Backus, Kehoe, and Kydland (1994), Blonigen and Wilson (1999), Corsetti, Dedola, and Leduc (2008), Enders, Müller, and Scholl (2011), or Heathcote and Perri (2002), among many others). Therefore, recognizing the difference between the functional forms to aggregate foreign products (as in this paper) is the key to understand the puzzle in the first place.

Second, compared to the existing literature, the overall investigation in this paper abstracts from the complications due to having a time dimension so that we can focus on the differences between the two literatures due to the way that they aggregate across foreign products. In contrast, studies such as by Ruhl (2008), Fitzgerald and Haller (2014) Ramanarayanan (2017) or Crucini and Davis (2016) all focus on solutions based on the difference between the two literatures due to the frictions created by the time dimension, such as uncertainties on productivities, the speed of adjustment of capital, or firm entry/exit decisions over time. However, since these studies do not recognize that the two literatures have distinct functional forms to aggregate across foreign products, their time-related frictions will only complicate the investigation, leading to improper comparisons between the two literatures.

The rest of the paper is organized as follows. The next section introduces demand-side models that are consistent with the two literatures. Section 3 attempts to understand the *international elasticity puzzle* by considering the importance of expenditure shares in CES frameworks with finite number of goods and countries. Section 4 provides empirical support for the theory introduced in this paper. Section 5 concludes. The Appendix shows the derivations of certain equations used in the main text.

2 The Economic Environment

This section introduces a model of international economics consisting of home and foreign countries. It is important to emphasize that the model and its implications are independent of the supply side, since such details are effectively eliminated during the comparison of international trade and international finance literatures (using their implications for the price elasticity of demand at the product level, as shown in the Appendix). Therefore, we only focus on the demand side until the empirical investigation below, where at least some structure is necessary on the supply side for estimation purposes. In terms of the notation, the superscripts represent the location of consumption, while the subscripts represent the location of production and goods.

2.1 Individuals

In most international finance studies, there are two tiers of aggregation representing the consumer utility: an upper-tier aggregation between home and foreign goods, and a lower-tier aggregation between foreign goods. In most international trade studies, there are also two tiers of aggregation: an upper-tier aggregation between countries of origins (including the home country), and a lower-tier aggregation between goods of each origin country. At the upper-tier aggregation (i.e., at the macro level), there is usually a *unique* foreign country (or an index of overall foreign/imported products) in international finance studies that is connected to the trade balance of the destination country, while there are *multiple* foreign countries/regions in international trade studies that are connected to bilateral trade of the destination country with each source country. Accordingly, although these literatures connect foreign products to home products by using an upper-tier aggregation, the number of foreign countries is different at the upper-tier aggregation, and, thus, there are alternative aggregation strategies of foreign products across the two literatures. Therefore, while products coming from alternative foreign countries are already connected to each other through the upper-tier aggregation in international trade, such alternative foreign countries are not distinguished between each other in international finance. In order to connect these two literatures, we consider an additional *middle-tier* aggregation across foreign countries while modeling individual utilities in international finance. The lower-tier aggregation is achieved across alternative goods coming from a particular country, which is the same between the two literatures, because the products coming from any country does not depend on how they are aggregated at the destination country.

In sum, in this paper, international trade individuals have two tiers of aggregation, representing source countries and goods, while international finance individuals have three tiers of aggregation, representing home versus foreign countries, foreign countries, and goods. It is assumed that there are finite number of goods and countries in both literatures. Further details of each literature are provided in the following subsections, while the definition of variables and parameters are given in Tables 2 and 3.

2.1.1 Individuals in International Trade

International trade studies such as by Anderson and Van Wincoop (2004), Head and Ries (2001), Hillberry and Hummels (2013), or Hummels (2001), among many others, have the following upper-tier CES aggregation, also called the Armington model as in Arkolakis, Costinot, and Rodríguez-Clare (2012), representing utility C^h in home country h:

$$C^h \equiv \left(\sum_i \left(\beta_i^h\right)^{\frac{1}{\theta}} \left(C_i^h\right)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \tag{1}$$

where C_i^h represents products coming from country i (which represents home products when $i = h$), θ is the elasticity of substitution across source countries (including both home and foreign countries) which we call as the *macro elasticity* in international trade, and β_i^h is a taste parameter (satisfying $\sum_i \beta_i^h = 1$). C_i^h is further given by the following lower-tier aggregation:

$$C_i^h \equiv \left(\sum_j \left(\beta_{ij}^h\right)^{\frac{1}{\eta}} \left(C_{ij}^h\right)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{2}$$

where C_{ij}^h represents good j coming from country i (which represents good j produced in home country when $i = h$), η is the elasticity of substitution across goods, and β_{ij}^h's represent taste parameters (satisfying $\sum_j \beta_{ij}^h = 1$). The optimal allocation of any given expenditure yields the following demand functions, where P_{ij}^h, P_i^h and P^h are the corresponding prices per units of C_{ij}^h, C_i^h and C^h, respectively:

$$C_{ij}^h = \beta_{ij}^h \left(\frac{P_{ij}^h}{P_i^h} \right)^{-\eta} C_i^h \tag{3}$$

and

$$C_i^h = \beta_i^h \left(\frac{P_i^h}{P^h} \right)^{-\theta} C^h \tag{4}$$

which can be utilized for the estimation of the macro elasticity θ in its log form using data on both international trade and intranational/domestic trade, since the latter is captured when $i = h$ (as we achieve in the empirical investigation, below). The last two expressions can be combined as follows:

$$C_{ij}^h = \beta_i^h \beta_{ij}^h \left(\frac{P_{ij}^h}{P_i^h} \right)^{-\eta} \left(\frac{P_i^h}{P^h} \right)^{-\theta} C^h \tag{5}$$

Finally, price indices are connected to each other through the standard expressions of:

$$P^h \equiv \left(\sum_i \beta_i^h \left(P_i^h\right)^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

and

$$P_i^h \equiv \left(\sum_j \beta_{ij}^h \left(P_{ij}^h\right)^{1-\eta} \right)^{\frac{1}{1-\eta}}$$

2.1.2 Individuals in International Finance

International finance studies such as by Backus, Kehoe, and Kydland (1994), Blonigen and Wilson (1999), Corsetti, Dedola, and Leduc (2008), Enders, Müller, and Scholl (2011), or Heathcote and Perri (2002), among many others, have the following upper-tier CES aggregation representing utility G^h in home country h:

$$G^h \equiv \left(\left(\alpha_h^h\right)^{\frac{1}{\sigma}} \left(C_h^h\right)^{\frac{\sigma-1}{\sigma}} + \left(1-\alpha_h^h\right)^{\frac{1}{\sigma}} \left(G_f^h\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{6}$$

where C_h^h (as in Equation 2 when $i = h$) and G_f^h represent home and foreign products, respectively, σ is the elasticity of substitution across home and foreign products which we call as the *macro elasticity* in international finance, and α_h^h is a taste parameter representing the preferences of individuals toward home products (that is related to home bias). Most international finance studies stop their disaggregation at this level which they use for their investigations. Nevertheless, it is understood that there is an additional tier of aggregation among foreign source countries for home country h (represented by $i \neq h$) in the background, which we achieve by putting more structure on the index of foreign products G_f^h as follows and call it the middle-tier aggregation:

$$G_f^h \equiv \left(\sum_{i \neq h} \left(\alpha_i^h\right)^{\frac{1}{\gamma}} \left(C_i^h\right)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}} \tag{7}$$

where C_i^h represents products imported from foreign country i as in the international trade literature above (as defined in Equation 2), because the products coming from foreign country i do not depend on how they are further aggregated at the destination country; γ is the elasticity of substitution across foreign countries, and α_i^h's (satisfying $\sum_{i \neq h} \alpha_i^h = 1$) represent source-specific taste parameters.

The lower-tier aggregation is assumed to be the same as in the international trade literature. Hence, the optimal allocation of any given expenditure yields the demand function in Equation 3 for this lower-tier aggregation, with the definitions of β_{ij}^h, P_{ij}^h, P_i^h and η being the same as in the international trade literature. Regarding the middle tier, the optimal allocation of any given expenditure yields the following demand function, where Q_f^h is the price per unit of G^h_f:

$$C^h_i = \alpha^h_i \left(\frac{P^h_i}{Q^h_f} \right)^{-\gamma} G^h_f \tag{8}$$

which can be utilized for the estimation of γ in its log form using data only on international trade (as we achieve in the empirical investigation, below). Regarding the upper tier, the optimal allocation of any given expenditure yields the following demand function, where Q^h is the price per unit of G^h:

$$G^h_f = \left(1 - \alpha^h_h\right) \left(\frac{Q^h_f}{Q^h} \right)^{-\sigma} G^h \tag{9}$$

which can be combined with Equations 3 and 8 to have the following demand function for good j coming from foreign country i:

$$C^h_{ij} = \left(1 - \alpha^h_h\right) \alpha^h_i \beta^h_{ij} \left(\frac{P^h_{ij}}{P^h_i} \right)^{-\eta} \left(\frac{P^h_i}{Q^h_f} \right)^{-\gamma} \left(\frac{Q^h_f}{Q^h} \right)^{-\sigma} G^h \tag{10}$$

Price indices are connected to each other through the standard expressions of:

$$Q^h_f \equiv \left(\sum_{i \neq h} \alpha^h_i \left(P^h_i\right)^{1-\gamma} \right)^{\frac{1}{1-\gamma}} \tag{11}$$

and

$$Q^h \equiv \left(\alpha^h_h \left(P^h_h\right)^{1-\sigma} + \left(1 - \alpha^h_h\right) \left(Q^h_f\right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{12}$$

Finally, the demand for good j produced at home is given by:

$$C^h_{hj} = \beta^h_{hj} \left(\frac{P^h_{hj}}{P^h_h} \right)^{-\eta} C^h_h \tag{13}$$

where

$$C^h_h = \alpha^h_h \left(\frac{P^h_h}{Q^h} \right)^{-\sigma} G^h \tag{14}$$

As is evident, as the (macro) elasticity of substitution across home and foreign products σ governs the substitutability between home and foreign products, it also shows up in the demand function for home products through their relative price with respect to the overall price index Q^h (that includes foreign prices according to Equation 12). We will use this information during our empirical investigation, below. The latter two expressions can be combined as follows:

$$C_{hj}^h = \alpha_h^h \beta_{hj}^h \left(\frac{P_{hj}^h}{P_h^h} \right)^{-\eta} \left(\frac{P_h^h}{Q^h} \right)^{-\sigma} G^h \tag{15}$$

where the price indices of home products P_h^h's are also the same across the two literatures, since they are independent of how foreign products are aggregated.

2.2 Price Elasticity of Demand

In any CES aggregation, the elasticity of substitution corresponds to the price elasticity of demand if the expenditure share of components in that aggregation is negligible (i.e., when there infinite number of components). However, this cannot be the case when there are finite number of components (as in this paper), especially when home and foreign products are aggregated, where the expenditure share of foreign products may not be negligible in a global world with high levels of trade openness. Accordingly, while calculating the price elasticity of demand in home country h, on top of the elasticity of substitution, we also consider the effects of disaggregated prices on aggregated price indices (due to having a finite number of goods and countries). The technical details of derivations are given in the Appendix.

In international trade, the price elasticity of demand for good j coming from country i (representing home products when $i = h$) is implied as follows:

$$-\frac{\partial C_{ij}^h}{\partial P_{ij}^h} \frac{P_{ij}^h}{C_{ij}^h} = \varepsilon \left(\eta, \theta, \omega_{ij}^h, \omega_i^h \right) = \eta \left(1 - \omega_{ij}^h \right) + \theta \omega_{ij}^h \left(1 - \omega_i^h \right) \tag{16}$$

where we have considered the effects of P_{ij}^h on aggregated price indices of P_i^h and P^h as well; $\omega_{ij}^h = \frac{P_{ij}^h C_{ij}^h}{P_i^h C_i^h}$ is the expenditure share of good j imported from country i among all products imported from country i, and $\omega_i^h = \frac{P_i^h C_i^h}{P^h C^h}$ is the expenditure share of country i products in the overall consumption.

As is evident, the price elasticity of demand in Equation 16 is a function of the macro elasticity in international trade θ, the elasticity of substitution across goods η, and expenditure weights. Compared to the existing literature, this expression is very similar to the one considered by Atkeson and Burstein (2008) in an alternative nested CES framework when price competition is considered across firms (i.e., Equation 19 in their paper), where an upper-tier aggregation is achieved in a continuum of sectors (which implies $\omega_i^h = 0$ for the upper-tier in this paper), and a lower-tier aggregation is achieved across a finite number of goods. Atkeson and Burstein (2008) have shown that considering finite number of goods (and thus good-specific market shares implying variable markups) is essential to explain why export and import prices show substantial and systematic deviations from relative PPP in comparison with source country producer prices. Different from Atkeson and Burstein (2008), the upper-tier aggregation in this paper is achieved across a finite number of foreign countries, and thus we have $\omega_i^h > 0$. Such an implication (of $\omega_i^h > 0$) is highly supported by the data used in this paper as well (see below for data details); e.g., within the U.S., the expenditure share of Kentucky on the products imported from Ohio is about 48% for the second quarter of 2012, while the expenditure share of Delaware on the products imported from New York is about 43% for the first quarter of 2012.

Similarly, in international finance, the price elasticity of demand for good j coming from foreign country i is implied as follows:

$$-\frac{\partial C_{ij}^h}{\partial P_{ij}^h}\frac{P_{ij}^h}{C_{ij}^h} = \varepsilon\left(\eta, \gamma, \sigma, \omega_{ij}^h, \chi_i^h, \chi_f^h\right) = \eta\left(1-\omega_{ij}^h\right) + \gamma\omega_{ij}^h\left(1-\chi_i^h\right) + \sigma\chi_i^h\omega_{ij}^h\left(1-\chi_f^h\right) \quad (17)$$

where we have considered the effects of P_{ij}^h on aggregated price indices of P_i^h, Q_f^h and Q^h as well; $\chi_i^h = \frac{P_i^h C_i^h}{Q_f^h G_f^h}$ (where $i \neq h$) is the expenditure share of country i products among all foreign products, and $\chi_f^h = \frac{Q_f^h G_f^h}{Q^h G^h} = 1 - \chi_h^h$ is the expenditure share of foreign products (i.e., imports) in the overall consumption, with $\chi_h^h = \omega_h^h$ representing the home expenditure share. As is evident, the price elasticity of demand is a function of the macro elasticity in international finance σ, the elasticity of substitution across foreign countries γ, the elasticity of substitution across goods η, and expenditure weights. As in the case of Equation 16, the implications provided in Equation 17 (of having sizable expenditure weights) are also consistent with the data used in this paper, where the state of Washington has a χ_i^h value of about 97% for the products coming from Oregon in the first quarter of 2012.

It is important to emphasize that both Equations 16 and 17 have the same definition (i.e., the price elasticity of demand for good j coming from foreign country i). Since this micro detail cannot be changed by the artificially created aggregation strategies in the two literatures, in the next Section, we equalize Equations 16 and 17 in order to have implications for the discrepancy between the two macro elasticity measures of θ and σ.

3 Implications for the International Elasticity Puzzle

We are interested in understanding the international elasticity puzzle regarding mixed policy implications. We consider the fact that the reactions of micro quantities (e.g., actual amount of goods traded in real life) to any changes affecting micro prices (e.g., good-level productivities, exchange rates, or tariff rates) cannot be affected by how they are aggregated at upper tiers of consumer utility, and thus they should be equalized across the two literatures. In other words, we would like to know the conditions under which the two literatures would provide the same policy implications. Accordingly, we equalize the price elasticity of demand measures for individual goods (coming from individual foreign countries) between international trade and international finance in this section. The technical details of derivations are given in the Appendix.

Since the left hand sides of Equations 16 and 17 have the same definition (i.e., the price elasticity of demand for good j coming from foreign country i), equalizing them to each other results in the following expression:

$$\theta = \gamma\Omega + \sigma\left(1 - \Omega\right) \tag{18}$$

where $\Omega = \frac{\sum_h (N_i - 1)}{\sum_h (N_i - 1) + \sum_h \chi_h^h}$ with N_i representing the number of foreign countries. As is evident, the macro elasticity in international trade θ is a weighted average of the macro elasticity in international finance σ and the elasticity of substitution across foreign countries γ (in international finance), where weights are determined by the number of foreign countries N_i and the home expenditure shares of χ_h^h's. It is easy to show that in a special case in which there is a unique foreign country under investigation (i.e., $N_i = 1$) as in most studies in international finance, it is implied that $\Omega = 0$ and thus the two literatures converge to each other regarding their macro elasticity measures as $\theta = \sigma$. However, as the number of foreign countries investigated increases as in most international trade studies, Ω gets higher and thus the two literatures diverge from each other as θ goes away from σ. Therefore, Equation 18 represents the general relationship between the macro elasticity measures of the two literatures based on the number of countries used in the investigation.

It is important to emphasize that the elasticity of substitution across goods η, together with the corresponding expenditure weights, has been effectively eliminated in Equation 18 (as shown in the Appendix) due to having the very same lower-tier aggregation (across goods of a foreign source country) in the two literatures. Therefore, the comparison between the macro elasticities of θ (coming from international trade) and σ (coming from international finance) does not depend on the value or the determination of η that governs the comparison across individual good characteristics that depend on how they are produced (and/or priced) in foreign source countries and/or how they are transported. It is implied that having a finite number of foreign countries (rather than a finite number of goods) is essential for a comparison of macro elasticities between the two literatures. It is also implied that independent of the supply side (e.g., having intermediate inputs or fixed production costs), or the aggregation of the varieties of each good (e.g., having a good-level or firm-level analysis), the results in this paper remain the same, since such micro-level details would effectively be eliminated during the comparison of the two literatures.[2]

In contrast, in the existing literature, Ruhl (2008) has proposed a solution to *the international elasticity puzzle* based on firm-level entry costs and uncertainties on future productivities in a Melitz (2003) framework; Fitzgerald and Haller (2014) have both fixed and sunk costs of export participation, where participation in different export markets are considered as independent decisions after conditioning on a common marginal cost of production; Crucini and Davis (2016) consider the speed of adjustment of capital in the distribution sector; Ramanarayanan (2017) considers intermediate inputs in which heterogeneous producers face a plant-level irreversibility in the structure of inputs used in production; Arkolakis, Eaton, and Kortum (2012) consider the difference between the adjustments in extensive and intensive margins of trade in an Eaton and Kortum (2002) framework. Note that all of these details are effectively eliminated in Equation 18 (as shown in the Appendix), because they are related to the good-level production stories that have nothing to do with how products of foreign source countries are aggregated through the macro elasticities of θ versus σ.

Overall, according to Equation 18, the policy implications are equalized between the two literatures by construction, although the artificially created upper-level elasticities θ, σ and γ are allowed to adjust to be consistent with this micro equality. The next section tests this implication empirically.

[2] As an example, one can easily (i) include an additional tier of CES aggregation across firms that disaggregates each foreign good imported from a source country, (ii) play with the supply side by including intermediate inputs, productivity differences, fixed costs, or (iii) include dynamic channels in the determination of good-level characteristics. However, when the micro-level price elasticities of demand are equalized between the two literatures, such micro-level details would be effectively eliminated as shown in the Appendix for the derivation of Equation 18.

4 Empirical Investigation

We have so far proposed a theoretical background to understand the international elasticity puzzle by showing that the macro elasticity in international trade θ is a weighted average of the macro elasticity in international finance σ and the elasticity of substitution across foreign countries in international finance γ (according to Equation 18). Hence, the conflicting estimates in the two literatures (as given in Table 1) can be understood by carefully considering the alternative definitions of foreign basket of goods. In this section, we will test this implication empirically, which requires the estimation of θ, γ and σ.

Since the objective is to make a comparison between the two literatures, some clarification is necessary regarding how the literature has estimated these elasticity measures. In particular, some international trade studies such as by Yotov (2012), Simonovska and Waugh (2014a) or Simonovska and Waugh (2014b) combine international trade with *ad hoc* measures of domestic trade obtained from the problematic national income and product accounts in their elasticity estimations, and thus their estimates correspond to θ in this paper. Obviously, these papers are different from those other international trade studies such as by Hummels (2001) or Head and Ries (2001) who use data on international trade only, and thus such estimates correspond to γ in this paper. Therefore, the international trade literature estimates the values of θ and γ interchangeably, meaning that both estimates of θ (obtained by the log version of Equation 4) and γ (obtained by the log version of Equation 8) can be compared with the existing literature depending on the data used in the estimation. Similarly, the macro elasticity in international finance σ can either be estimated by using the log version of 9 or the log version of Equation 14; due to data availability, we consider the latter option in our empirical investigation below.

4.1 Data and Estimation Methodology

We use Equations 4, 8 and 14 for the estimation of θ, γ and σ, respectively, which require data on both domestic and foreign trade. Since international trade data sources do not collect data on domestic trade, rather than employing the problematic domestic trade values implied by national income and product accounts, we shift our focus to interstate trade data within the U.S. obtained from the Commodity Flow Survey (CFS) for the year of 2012, which provide quarterly export data on both *interstate* (foreign) trade and *intrastate* (domestic) trade.[3] Since we use "2012 CFS Public Use Microdata File" obtained from the web page of the U.S. Census Bureau, following Hillberry and Hummels (2003), we are able to eliminate the problematic observations of wholesale and retail shipments and focus only on mining and manufacturing shipments across states.[4] As in studies such as by Anderson and Van Wincoop (2003) and Anderson and Van Wincoop (2004), we identify θ, γ and σ through the coefficients in front of log distance, subject to the determination of the distance elasticity of trade costs. Accordingly, we only consider transportation through private trucks across states so that having alternative transportation modes that are used less (e.g., water, rail or air) does not affect the estimated coefficients. Our CFS data set also provides information on the actual distance of shipment between (or within) states that changes (due to alternative shipment routes that consider the exact source and destination locations within each state) across quarters of 2012.[5]

The estimation requires information on prices which can be achieved by the details of the supply side. In order to keep the supply side as simple as possible, we only assume the following relationship between source and destination prices for good j:

$$P_{ijt}^{h} = \tau_{it}^{h} P_{ijt}^{h*} \tag{19}$$

where the subscript t represents quarterly nature of the trade data, $\tau_{it}^{h} > 1$ represents (gross) iceberg trade costs, and P_{ijt}^{h*} represents source prices given by:

$$P_{ijt}^{h*} = \frac{w_i}{z_{ij} \exp(\rho t)} \tag{20}$$

where w_i represents a source-specific cost of production that is steady over the quarters of 2012 (e.g., sticky wages), z_{ij} represents the level of productivity, and $\exp(\rho t)$ captures changes in productivity over time that are common across goods and states. Since trade data are from within the U.S., we further approximate time-varying trade costs with $\tau_{it}^{h} = \left(d_{it}^{h}\right)^{\delta}$, where d_{it}^{h} represents quarter-specific distance between states i and h (because it represents the actual

[3]This is well-known data source that has also been used in earlier studies such as by Wolf (2000), Hillberry and Hummels (2003) and.Yilmazkuday (2012) for alternative years.

[4]The final trade data used in the estimation include observations on North American Industry Classification System (NAICS) industry codes of 212, 311, 312, 313, 314, 315, 316, 321, 322, 323, 324, 325, 326, 327, 331, 332, 333, 334, 335, 336, 337, 339.

[5]See Yilmazkuday (2014) for the importance of considering the exact location of shipments in the estimation of distance coefficients.

shipment distance that may change each quarter), and δ is the distance elasticity of trade costs.

For the estimation of θ, we use the log version of Equation 4 with the addition of the time dimension (due to using quarterly data) as follows:

$$\log\left(P_{it}^{h}C_{it}^{h}\right)=(1-\theta)\log P_{it}^{h}+\log\left(\left(P_{t}^{h}\right)^{\theta}C_{t}^{h}\right)+\log\beta_{it}^{h} \tag{21}$$

where $P_{it}^{h}\equiv\left(\sum_{j}\beta_{ijt}^{h}\left(P_{ijt}^{h}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}$ can be rewritten according to Equation 19 as $P_{it}^{h}\equiv\left(d_{it}^{h}\right)^{\delta}P_{it}^{h*}=\left(d_{it}^{h}\right)^{\delta}w_{i}\exp\left(\rho t\right)\varsigma_{it}^{h}$, P_{it}^{h*} represents P_{it}^{h} measured at the source, and $\varsigma_{it}^{h}=\left(\sum_{j}\beta_{ijt}^{h}\left(z_{ij}\right)^{\eta-1}\right)^{\frac{1}{1-\eta}}$. When quarterly exports data are used (i.e., when trade is measured at the source as in this paper), according to $P_{it}^{h}\equiv\left(d_{it}^{h}\right)^{\delta}P_{it}^{h*}$, it is implied that:

$$\underbrace{\log\left(P_{it}^{h*}C_{it}^{h}\right)}_{\text{Quarterly Trade Data}}=-\delta\theta\underbrace{\log d_{it}^{h}}_{\text{Quarterly Distance}}+\underbrace{(1-\theta)\log w_{i}}_{\text{Source Fixed Effects}}+\underbrace{\log\left(\left(P^{h}\right)^{\theta}C^{h}\right)}_{\text{Destination Fixed Effects}} \tag{22}$$

$$+\underbrace{\rho t}_{\text{Quarter Fixed Effects}}+\underbrace{(1-\theta)\log\varsigma_{it}^{h}\beta_{it}^{h}}_{\text{Residuals}}$$

where the only time-varying variables are the quarterly exports data on the left hand side together with time-varying distance measures (due to alternative actual distance measures of shipment across quarters) and preferences (captured by residuals) on the right hand side. Since we have bilateral panel data, the identification of $-\theta\delta$ is achieved through both time and dyadic cross-sectional dimensions of the data as in the international trade literature. It is important to note that the trade data in this estimation include both interstate and intrastate trade according to Equation 1.

Similarly, for the estimation of γ with quarterly exports data (measured at the source), we use the log version of Equation 8 combined with Equations 19 and 20 as follows:

$$\underbrace{\log\left(P_{it}^{h*}C_{it}^{h}\right)}_{\text{Quarterly Interstate Trade Data}}=-\gamma\delta\underbrace{\log d_{it}^{h}}_{\text{Quarterly Distance}}+\underbrace{(1-\gamma)\log w_{i}}_{\text{Source Fixed Effects}}+\underbrace{\log\left(\left(Q_{f}^{h}\right)^{\gamma}G_{f}^{h}\right)}_{\text{Destination Fixed Effects}} \tag{23}$$

$$+\underbrace{\rho t}_{\text{Quarter Fixed Effects}}+\underbrace{(1-\gamma)\log\varsigma_{it}^{h}\alpha_{it}^{h}}_{\text{Residuals}}$$

where the only difference with respect to Equation 22 is that the trade data only include interstate trade (and exclude intrastate trade) according to Equation 7. Since we have bilateral panel data, the identification of $-\gamma\delta$ is again achieved through both time and dyadic cross-sectional dimensions of the data as in the international trade literature.

Finally, for the estimation of σ with quarterly exports data (measured at the source), we use the log version of Equation 14 combined with Equations 19 and 20 as follows:

$$\underbrace{\log\left(P_{ht}^{h}C_{ht}^{h}\right)}_{\text{Quarterly Intrastate Trade Data}} = -\sigma\delta \underbrace{\log d_{ht}^{h}}_{\text{Quarterly Distance}} + \underbrace{\log\left(\left(w_{h}\right)^{1-\sigma}\left(Q^{h}\right)^{\sigma}G^{h}\right)}_{\text{State Fixed Effects}} + \underbrace{\rho t}_{\text{Quarter Fixed Effects}} + \underbrace{(1-\sigma)\log\varsigma_{ht}^{h}\alpha_{ht}^{h}}_{\text{Residuals}} \tag{24}$$

where only intrastate trade data are used. Since the panel data are at the state level for this estimation, the identification of $-\sigma\delta$ is achieved through the time and cross-sectional dimensions of the data as in the international finance literature.

Overall, the estimations of $\theta\delta$, $\gamma\delta$ and $\sigma\delta$ are simply achieved by changing the way that the observations are pooled across states, while the right hand side variables are technically the same. In particular, $\theta\delta$ can be estimated when data on both interstate and intrastate trade are pooled, $\gamma\delta$ can be estimated when data on only *interstate* trade are used, and $\sigma\delta$ can be estimated when data on only *intrastate* trade are used. After the estimations are achieved, elasticity measures of θ, γ and σ can further be identified for any given δ measure that can be borrowed from the literature, as also achieved by studies such as by Anderson and Van Wincoop (2003) or Anderson and Van Wincoop (2004). It is important to emphasize that the comparison between the relative values of θ, γ and σ (and thus the comparison of the two literatures regarding their macro implications) do not depend on the value of δ, since it is their multiplication with the same δ (i.e., $-\delta\theta$, $-\gamma\delta$ and $-\sigma\delta$) that is estimated as the coefficient in front of log distance in Equations 22, 23 and 24.

4.2 Empirical Results

The estimation results are given in Table 4, where all estimates of $\theta\delta$, $\gamma\delta$ and $\sigma\delta$ are statistically significant and have their expected signs. As is evident, for any given value of δ, the estimated macro elasticity measures in international trade (represented by θ or γ) are higher than those in international finance (represented by σ). Therefore, the existence of the international elasticity puzzle is confirmed by the empirical results of this paper, independent of δ.

Regarding the main objective of our empirical investigation, the estimation results in Table 4 also confirm Equation 18 in the sense that θ is a weighted average γ and σ, where Ω represents the weight based on the number of foreign countries and the home expenditure share. In particular, according to Equation 18, this weight Ω can be rewritten as follows:

$$\Omega = \frac{\theta - \sigma}{\gamma - \sigma} = \frac{\theta\delta - \sigma\delta}{\gamma\delta - \sigma\delta} \tag{25}$$

where the second equality has been obtained by multiplying both the numerator and the denominator by δ, in order to show that our results are independent of the value of δ.

According to the estimates of $\theta\delta$, $\gamma\delta$ and $\sigma\delta$ in Table 4, an estimate of $\Omega = 0.881$ is implied with a 95% confidence interval (calculated by the Delta method) between 0.763 and 0.996. When we directly use the theory-implied definition of Ω in Equation 18, we obtain a value of 0.937 by using home expenditure shares χ_h^h's and the number of foreign countries N_i (that correspond to the number of interstate trade partners in our empirical investigation) for each state.[6] Therefore, the theory-implied measure of $\Omega = 0.937$ lies within the 95% confidence interval of the estimated measure of Ω (obtained by the estimated values of $\theta\delta$, $\gamma\delta$ and $\sigma\delta$ in Table 4). It is implied that independent of the value of δ, θ is in fact a weighted average γ and σ, and the weights are determined by the number of foreign countries and home expenditure shares as indicated by Equation 18. Therefore, the equalization of price elasticity measures is enough to understand the discrepancy between the macro elasticity measures in the two literatures, both theoretically and empirically.

In order to show the contribution of this paper in a clear way, consider the opposite case in which the implications of a finite number of foreign countries are not considered (and thus expenditure weights are negligible in the calculation of price elasticity measures). In such a case, we would have $\chi_h^h = 0$ for all h, which would imply that $\Omega = 1$ and thus $\theta = \gamma$ according to Equation 18, suggesting that there is no connection between the two literatures regarding their macro elasticity measures of θ and σ. Apparently, such a restrictive assumption is

[6] In this calculation, we use the measures of $\sum_h (N_i - 1) = 1938$ and $\sum_h \chi_h^h = 130.345$ that are obtained from the very same data used in the estimation.

inconsistent with not only solving the international elasticity puzzle theoretically but also the estimated value of $\Omega = 0.881$ (with a 95% confidence interval between 0.763 and 0.996) empirically. Therefore, in technical terms, it is essential to consider a finite number of countries (and thus positive expenditure weights entering price elasticity calculations) to understand the international elasticity puzzle.

To further check the consistency of these empirical results with those in the literature, we would also like to work on the actual estimates of θ, γ and σ. Accordingly, we need a measure of δ in our investigation. We borrow the value of $\delta = 0.45$ from Yilmazkuday (2012), with a wide range between 0.18 and 0.84 (for robustness purposes) that also covers the estimate of $\delta = 0.38$ by Limao and Venables (2001) obtained by using actual shipping company quotes as well as using international trade costs data.[7] Based on $\delta = 0.45$, the implied elasticity estimates of θ, γ and σ are also given in Table 4.[8] While the elasticity estimate of $\theta = 4.875$ (with a 95% confidence interval between 2.508 and 12.672) is highly consistent with the international trade literature (as detailed in Table 1), the elasticity estimate of $\sigma = 0.615$ (with a 95% confidence interval between 0.020 and 2.984) is highly consistent with the international finance literature (also as detailed in Table 1). Therefore, our empirical investigation not only helps us understand the international elasticity puzzle but also results in elasticity estimates that are highly consistent with the existing literature on both international trade and international finance.

5 Conclusion

International trade studies have higher macro elasticity measures compared to international finance studies. This observation has been puzzling for many researchers due to the fact that price changes (that are caused by changes in international finance measures such as exchange rates or by changes in international trade measures such as trade costs) are transferred into quantity and thus welfare changes through these elasticities, and having alternative elasticity measures corresponds to alternative policy implications (as in Ruhl (2008)).

[7] Yilmazkuday (2012) finds the value of $\delta = 0.45$ (with a range between 0.18 and 0.84 across industries) as the average estimate across NAICS industries obtained by using interstate trade data (coming from Commodity Flow Survey) within the U.S. for the year of 2007.

[8] An alternative estimate of $\delta = 0.38$ has been found by Limao and Venables (2001) using actual shipping company quotes, but this value would not change any of our empirical results, below, since it is within the range of $\delta \in [0.18, 0.84]$ that we consider in our calculations.

This paper has been an attempt to understand the discrepancy between the macro elasticity measures in the two literatures by carefully taking into account their alternative strategies used in the aggregation of foreign products. Accordingly, the conditions under which the two literatures imply the very same policy implications have been investigated. In nested CES frameworks with a finite number of foreign countries (that are consistent with the existing studies), this has been achieved by equalizing the price elasticity of demand measures between the two literatures. Such an approach has resulted in an expression that explains the discrepancy in the macro elasticity measures. In particular, it has been theoretically shown that the macro elasticity in international trade is a weighted average of the macro elasticity in international finance and the elasticity of substitution across products of different foreign source countries. An empirical investigation has further supported this theoretical result in a clear way.

Regarding policy implications (e.g., effects of an exchange-rate change in international finance versus a trade-cost change in international trade, both through destination prices), they are equalized in this paper between the two literatures by construction due to equalizing the price elasticity of demand measures, although the artificially created upper-level elasticities are allowed to adjust to be consistent with this equality. It is implied that any policy analysis should be achieved at the disaggregated (bilateral-country) level and then aggregated up to obtain macro implications for the destination country; otherwise, the upper-tier macro elasticity measures can be misleading (and thus result in the international elasticity puzzle) due to the way that foreign products are aggregated at the destination country. Finally, since the supply-side details are cancelled out during the comparison of the price elasticity of demand measures between the two literatures, the theoretical results in this paper are independent of the supply-side (e.g., having intermediate inputs or fixed production costs) or the aggregation of the varieties of each good (e.g., having a good-level or firm-level analysis).

References

Anderson, J. E., and E. Van Wincoop (2003): "Gravity with gravitas: a solution to the border puzzle," *The American Economic Review*, 93(1), 170–192.

——— (2004): "Trade costs," *Journal of Economic literature*, 42(3), 691–751.

Arkolakis, C., A. Costinot, and A. Rodríguez-Clare (2012): "New trade models, same old gains?," *The American Economic Review*, 102(1), 94–130.

Arkolakis, C., J. Eaton, and S. Kortum (2012): "Staggered adjustment and trade dynamics," Discussion paper, mimeo.

Atkeson, A., and A. Burstein (2008): "Pricing-to-market, trade costs, and international relative prices," *American Economic Review*, 98(5), 1998–2031.

Backus, D. K., P. J. Kehoe, and F. E. Kydland (1994): "Dynamics of the Trade Balance and the Terms of Trade: The J-Curve?," *The American Economic Review*, 84(1), 84–103.

Bergin, P. R. (2006): "How well can the New Open Economy Macroeconomics explain the exchange rate and current account?," *Journal of International Money and Finance*, 25, 675–701.

Blonigen, B. A., and W. W. Wilson (1999): "Explaining Armington: what determines substitutability between home and foreign goods?," *Canadian Journal of Economics*, pp. 1–21.

Clausing, K. A. (2001): "Trade creation and trade diversion in the Canada–United States free trade agreement," *Canadian Journal of Economics/Revue canadienne d'économique*, 34(3), 677–696.

Corsetti, G., L. Dedola, and S. Leduc (2008): "High exchange-rate volatility and low pass-through," *Journal of Monetary Economics*, 55(6), 1113–1128.

Corsetti, G., and P. Pesenti (2001): "Welfare and Macroeconomic Interdependence," *The Quarterly Journal of Economics*, 116(2), 421–445.

Crucini, M. J., and J. S. Davis (2016): "Distribution capital and the short-and long-run import demand elasticity," *Journal of International Economics*, 100, 203–219.

Eaton, J., and S. Kortum (2002): "Technology, geography, and trade," *Econometrica*, 70(5), 1741–1779.

Enders, Z., G. J. Müller, and A. Scholl (2011): "How do fiscal and technology shocks affect real exchange rates?: New evidence for the United States," *Journal of International Economics*, 83(1), 53–69.

Feenstra, R. C., P. Luck, M. Obstfeld, and K. N. Russ (2018): "In search of the Armington elasticity," *Review of Economics and Statistics*, 100(1), 135–150.

Fitzgerald, D., and S. Haller (2014): "Exporters and shocks: Dissecting the international elasticity puzzle," Discussion paper, National Bureau of Economic Research.

Gali, J., and T. Monacelli (2005): "Monetary policy and exchange rate volatility in a small open economy," *The Review of Economic Studies*, 72(3), 707–734.

Head, K., and J. Ries (2001): "Increasing returns versus national product differentiation as an explanation for the pattern of US-Canada trade," *American Economic Review*, pp. 858–876.

Heathcote, J., and F. Perri (2002): "Financial autarky and international business cycles," *Journal of Monetary Economics*, 49(3), 601–627.

Hillberry, R., and D. Hummels (2003): "Intranational home bias: Some explanations," *Review of Economics and Statistics*, 85(4), 1089–1092.

——— (2013): "Trade elasticity parameters for a computable general equilibrium model," *Handbook of Computable General Equilibrium Modeling*, 1, 1213–1269.

Hummels, D. (2001): "Toward a Geography of Trade Costs," *mimeo.*

Limao, N., and A. J. Venables (2001): "Infrastructure, geographical disadvantage, transport costs, and trade," *The World Bank Economic Review*, 15(3), 451–479.

Melitz, M. J. (2003): "The impact of trade on intra-industry reallocations and aggregate industry productivity," *Econometrica*, 71(6), 1695–1725.

Ramanarayanan, A. (2017): "Imported inputs, irreversibility, and international trade dynamics," *Journal of International Economics*, 104, 1–18.

Ruhl, K. J. (2008): "The international elasticity puzzle," *unpublished paper, NYU.*

Simonovska, I., and M. E. Waugh (2014a): "The elasticity of trade: Estimates and evidence," *Journal of International Economics*, 92(1), 34–50.

——— (2014b): "Trade models, trade elasticities, and the gains from trade," Discussion paper, National Bureau of Economic Research.

Stockman, A. C., and L. L. Tesar (1995): "Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements," *The American Economic Review*, pp. 168–185.

Wolf, H. C. (2000): "Intranational home bias in trade," *Review of economics and statistics*, 82(4), 555–563.

Yilmazkuday, H. (2012): "Understanding interstate trade patterns," *Journal of International Economics*, 86(1), 158–166.

——— (2014): "Mismeasurement of distance effects: The role of internal location of production," *Review of International Economics*, 22(5), 992–1015.

Yotov, Y. V. (2012): "A simple solution to the distance puzzle in international trade," *Economics Letters*, 117(3), 794–798.

6 Appendix

6.1 Derivation of the Price Elasticity of Demand Measures

This Appendix depicts the derivations of Equations 16 and 17 in the main text.

6.1.1 Price Elasticity of Demand in International Trade

The demand for C_{ij}^h representing good j coming from country i in international trade is given by Equation 5 as follows:

$$C_{ij}^h = \beta_i^h \beta_{ij}^h \left(\frac{P_{ij}^h}{P_i^h} \right)^{-\eta} \left(\frac{P_i^h}{P^h} \right)^{-\theta} C^h \qquad (26)$$

The corresponding price elasticity of demand is calculated by considering the effects of P_{ij}^h on P_i^h and P^h. Accordingly, we start with rewriting the demand function as follows:

$$C_{ij}^h = \beta_i^h \beta_{ij}^h \left(P_{ij}^h\right)^{-\eta} \left(P_i^h\right)^{\eta-\theta} \left(P^h\right)^{\theta} C^h \tag{27}$$

By using the product rule, the price elasticity of demand can be written as follows:

$$-\frac{\partial C_{ij}^h}{\partial P_{ij}^h}\frac{P_{ij}^h}{C_{ij}^h} = -\left(\begin{array}{c} \frac{\partial\left(\beta_i^h\beta_{ij}^h\left(P_{ij}^h\right)^{-\eta}\right)}{\partial P_{ij}^h}\left(\left(P_i^h\right)^{\eta-\theta}\left(P^h\right)^{\theta}C^h\right) \\ \\ +\frac{\partial\left(\left(P_i^h\right)^{\eta-\theta}\left(P^h\right)^{\theta}C^h\right)}{\partial P_{ij}^h}\left(\beta_i^h\beta_{ij}^h\left(P_{ij}^h\right)^{-\eta}\right) \end{array}\right)\frac{P_{ij}^h}{C_{ij}^h} \tag{28}$$

where

$$\frac{\partial\left(\beta_i^h\beta_{ij}^h\left(P_{ij}^h\right)^{-\eta}\right)}{\partial P_{ij}^h} = \frac{-\eta\beta_i^h\beta_{ij}^h\left(P_{ij}^h\right)^{-\eta}}{P_{ij}^h} \tag{29}$$

The product rule can be used one more time for the derivative in the second component within the parenthesis as follows:

$$\frac{\partial\left(\left(P_i^h\right)^{\eta-\theta}\left(P^h\right)^{\theta}C^h\right)}{\partial P_{ij}^h} = \frac{\partial\left(\left(P_i^h\right)^{\eta-\theta}\right)}{\partial P_{ij}^h}\left(P^h\right)^{\theta}C^h + \frac{\partial\left(\left(P^h\right)^{\theta}\right)}{\partial P_{ij}^h}\left(P_i^h\right)^{\eta-\theta}C^h$$

where $P^h \equiv \left(\sum_i \beta_i^h \left(P_i^h\right)^{1-\theta}\right)^{\frac{1}{1-\theta}}$ and $P_i^h \equiv \left(\sum_j \beta_{ij}^h \left(P_{ij}^h\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}$ as given in the main text. Substituting these price expressions into this derivative, we obtain:

$$\frac{\partial\left(\left(P_i^h\right)^{\eta-\theta}\left(P^h\right)^{\theta}C^h\right)}{\partial P_{ij}^h} = \frac{\partial\left(\left(\left(\sum_j \beta_{ij}^h\left(P_{ij}^h\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}\right)^{\eta-\theta}\right)}{\partial P_{ij}^h}\left(P^h\right)^{\theta}C^h$$

$$+\frac{\partial\left(\left(\left(\sum_i \beta_i^h\left(\left(\sum_j \beta_{ij}^h\left(P_{ij}^h\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}\right)^{1-\theta}\right)^{\frac{1}{1-\theta}}\right)^{\theta}\right)}{\partial P_{ij}^h}\left(P_i^h\right)^{\eta-\theta}C^h$$

which can be rewritten by using the power and chain rules as follows:

$$\frac{\partial\left(\left(P_i^h\right)^{\eta-\theta}\left(P^h\right)^{\theta}C^h\right)}{\partial P_{ij}^h}=\left(\frac{(\eta-\theta)\left(\sum_j\beta_{ij}^h\left(P_{ij}^h\right)^{1-\eta}\right)^{\frac{\eta-\theta}{1-\eta}}}{\left(\sum_j\beta_{ij}^h\left(P_{ij}^h\right)^{1-\eta}\right)}\frac{\beta_{ij}^h\left(P_{ij}^h\right)^{1-\eta}}{P_{ij}^h}\right)\left(P^h\right)^{\theta}C^h$$

$$+\left(\begin{array}{c}\theta\frac{\left(\sum_i\beta_i^h\left(\left(\sum_j\beta_{ij}^h\left(P_{ij}^h\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}\right)^{1-\theta}\right)^{\frac{\theta}{1-\theta}}}{\left(\sum_i\beta_i^h\left(\left(\sum_j\beta_{ij}^h\left(P_{ij}^h\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}\right)^{1-\theta}\right)}\beta_i^h\frac{\left(\left(\sum_j\beta_{ij}^h\left(P_{ij}^h\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}\right)^{1-\theta}}{\left(\left(\sum_j\beta_{ij}^h\left(P_{ij}^h\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}\right)}\\ \times\frac{\left(\sum_j\beta_{ij}^h\left(P_{ij}^h\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}}{\left(\sum_j\beta_{ij}^h\left(P_{ij}^h\right)^{1-\eta}\right)}\beta_{ij}^h\frac{\left(P_{ij}^h\right)^{1-\eta}}{P_{ij}^h}\left(\left(P_i^h\right)^{\eta-\theta}C^h\right)\end{array}\right)$$

Substituting $P^h\equiv\left(\sum_i\beta_i^h\left(P_i^h\right)^{1-\theta}\right)^{\frac{1}{1-\theta}}$ and $P_i^h\equiv\left(\sum_j\beta_{ij}^h\left(P_{ij}^h\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}$ back into this equation results in:

$$\frac{\partial\left(\left(P_i^h\right)^{\eta-\theta}\left(P^h\right)^{\theta}C^h\right)}{\partial P_{ij}^h}=\left(\frac{(\eta-\theta)\left(P_i^h\right)^{\eta-\theta}}{\left(P_i^h\right)^{1-\eta}}\frac{\beta_{ij}^h\left(P_{ij}^h\right)^{1-\eta}}{P_{ij}^h}\right)\left(P^h\right)^{\theta}C^h$$

$$+\left(\theta\frac{\left(P^h\right)^{\theta}}{\left(P^h\right)^{1-\theta}}\beta_i^h\frac{\left(P_i^h\right)^{1-\theta}}{P_i^h}\frac{P_i^h}{\left(P_i^h\right)^{1-\eta}}\beta_{ij}^h\frac{\left(P_{ij}^h\right)^{1-\eta}}{P_{ij}^h}\left(\left(P_i^h\right)^{\eta-\theta}C^h\right)\right)$$

Using the expenditure shares obtained from the demand functions given in Equations 3 and 4, which are:

$$\omega_{ij}^h=\frac{P_{ij}^hC_{ij}^h}{P_i^hC_i^h}=\beta_{ij}^h\left(\frac{P_{ij}^h}{P_i^h}\right)^{1-\eta}\tag{30}$$

and

$$\omega_i^h=\frac{P_i^hC_i^h}{P^hC^h}=\beta_i^h\left(\frac{P_i^h}{P^h}\right)^{1-\theta}\tag{31}$$

we can rewrite $\frac{\partial\left(\left(P_i^h\right)^{\eta-\theta}\left(P^h\right)^{\theta}C^h\right)}{\partial P_{ij}^h}$ as follows:

$$\frac{\partial\left(\left(P_i^h\right)^{\eta-\theta}\left(P^h\right)^{\theta}C^h\right)}{\partial P_{ij}^h}=\left((\eta-\theta)\,\omega_{ij}^h+\theta\omega_i^h\omega_{ij}^h\right)\left(\frac{\left(P_i^h\right)^{\eta-\theta}}{P_{ij}^h}\right)\frac{P^hC^h}{\left(P^h\right)^{1-\theta}}\tag{32}$$

Hence, the price elasticity of demand can be written as follows using Equations 29 and 32:

$$-\frac{\partial C_{ij}^h}{\partial P_{ij}^h}\frac{P_{ij}^h}{C_{ij}^h} = -\left(\begin{array}{c} \frac{-\eta\beta_i^h\beta_{ij}^h\left(P_{ij}^h\right)^{-\eta}}{P_{ij}^h}\left(\left(P_i^h\right)^{\eta-\theta}\left(P^h\right)^{\theta}C^h\right) \\ +\left(\left(\eta-\theta\right)\omega_{ij}^h+\theta\omega_i^h\omega_{ij}^h\right)\left(\frac{\left(P_i^h\right)^{\eta-\theta}}{P_{ij}^h}\right)\frac{P^hC^h}{\left(P^h\right)^{1-\theta}}\left(\beta_i^h\beta_{ij}^h\left(P_{ij}^h\right)^{-\eta}\right) \end{array}\right)\frac{P_{ij}^h}{C_{ij}^h} \quad (33)$$

which can be rewritten using Equations 30 and 31 as follows:

$$-\frac{\partial C_{ij}^h}{\partial P_{ij}^h}\frac{P_{ij}^h}{C_{ij}^h} = \varepsilon\left(\eta,\theta,\omega_{ij}^h,\omega_i^h\right) = \eta\left(1-\omega_{ij}^h\right)+\theta\omega_{ij}^h\left(1-\omega_i^h\right) \quad (34)$$

which is Equation 16 in the main text.

6.1.2 Price Elasticity of Demand in International Finance

The demand for C_{ij}^h representing good j coming from country i in international finance is given by Equation 10 as follows:

$$C_{ij}^h = \left(1-\alpha_h^h\right)\alpha_i^h\beta_{ij}^h\left(\frac{P_{ij}^h}{P_i^h}\right)^{-\eta}\left(\frac{P_i^h}{Q_f^h}\right)^{-\gamma}\left(\frac{Q_f^h}{Q^h}\right)^{-\sigma}G^h \quad (35)$$

The corresponding price elasticity of demand is calculated by considering the effects of P_{ij}^h on P_i^h, Q_f^h and Q^h. Accordingly, we start with rewriting the demand function as follows:

$$C_{ij}^h = \left(1-\alpha_h^h\right)\alpha_i^h\beta_{ij}^h\left(P_{ij}^h\right)^{-\eta}\left(P_i^h\right)^{\eta-\gamma}\left(Q_f^h\right)^{\gamma-\sigma}\left(Q^h\right)^{\sigma}G^h \quad (36)$$

The derivation strategy is exactly the same as in the case of international trade (above), with the additional expenditure share definitions obtained from Equations 8 and 9:

$$\chi_i^h = \frac{P_i^hC_i^h}{Q_f^hG_f^h} = \alpha_i^h\left(\frac{P_i^h}{Q_f^h}\right)^{1-\gamma} \quad (37)$$

and

$$\chi_f^h = 1-\chi_h^h = \frac{Q_f^hG_f^h}{Q^hG^h} = \left(1-\alpha_h^h\right)\left(\frac{Q_f^h}{Q^h}\right)^{1-\sigma} \quad (38)$$

Using the same derivation strategy (i.e., using product, power, and chain rules) as in the previous subsection results in the following expression:

$$-\frac{\partial C_{ij}^h}{\partial P_{ij}^h}\frac{P_{ij}^h}{C_{ij}^h} = \varepsilon\left(\eta, \gamma, \sigma, \omega_{ij}^h, \chi_i^h, \chi_f^h\right) = \eta\left(1-\omega_{ij}^h\right) + \gamma\omega_{ij}^h\left(1-\chi_i^h\right) + \sigma\chi_i^h\omega_{ij}^h\left(1-\chi_f^h\right) \quad (39)$$

which is Equation 17 in the main text.

6.2 Equalizing the Price Elasticity of Demand Measures

This Appendix depicts the derivation of Equation 18 in the main text. Since the left hand sides of Equations 16 and 17 have the same definition (i.e., the price elasticity of demand for good j coming from foreign country i), equalizing them to each other is achieved as follows:

$$-\frac{\partial C_{ij}^h}{\partial P_{ij}^h}\frac{P_{ij}^h}{C_{ij}^h} = \varepsilon\left(\eta, \theta, \omega_{ij}^h, \omega_i^h\right) = \varepsilon\left(\eta, \gamma, \sigma, \omega_{ij}^h, \chi_i^h, \chi_f^h\right)$$

which implies according to Equations 16 and 17 that:

$$\theta\omega_{ij}^h\left(1-\omega_i^h\right) = \gamma\omega_{ij}^h\left(1-\chi_i^h\right) + \sigma\chi_i^h\omega_{ij}^h\left(1-\chi_f^h\right)$$

where $\eta\left(1-\omega_{ij}^h\right)$ is effectively eliminated from both sides. After dividing both sides by ω_{ij}^h, considering the fact that the total expenditure E^h is the same between the two literatures (i.e., $E^h = P^hC^h = Q^hG^h$, regardless of the aggregation strategy), and using the expenditure share definitions given in Equations 30, 31, 37 and 38, we can rewrite this equality as follows:

$$\theta\left(1-\frac{P_i^hC_i^h}{Q_f^hG_f^h}\frac{Q_f^hG_f^h}{Q^hG^h}\right) = \gamma\left(1-\frac{P_i^hC_i^h}{Q_f^hG_f^h}\right) + \sigma\left(\frac{P_i^hC_i^h}{Q_f^hG_f^h} - \frac{P_i^hC_i^h}{Q_f^hG_f^h}\frac{Q_f^hG_f^h}{Q^hG^h}\right)$$

Since all the values within the parentheses are source country i and home/destination country h specific, in order to have an expression that would hold for all countries, we take the sum across all foreign countries (represented by $i \neq h$) and home countries (represented by h) as follows:

$$\theta\sum_h\sum_{i\neq h}\left(1-\frac{P_i^hC_i^h}{Q_f^hG_f^h}\frac{Q_f^hG_f^h}{Q^hG^h}\right) = \gamma\sum_h\sum_{i\neq h}\left(1-\frac{P_i^hC_i^h}{Q_f^hG_f^h}\right) + \sigma\sum_h\sum_{i\neq h}\left(\frac{P_i^hC_i^h}{Q_f^hG_f^h} - \frac{P_i^hC_i^h}{Q_f^hG_f^h}\frac{Q_f^hG_f^h}{Q^hG^h}\right)$$

which can be rewritten as follows:

$$\theta\left(\sum_h\sum_{i\neq h}1-\sum_h\sum_{i\neq h}\frac{P_i^hC_i^h}{Q_f^hG_f^h}\frac{Q_f^hG_f^h}{Q^hG^h}\right)=\gamma\left(\sum_h\sum_{i\neq h}1-\sum_h\sum_{i\neq h}\frac{P_i^hC_i^h}{Q_f^hG_f^h}\right)$$

$$+\,\sigma\left(\sum_h\sum_{i\neq h}\frac{P_i^hC_i^h}{Q_f^hG_f^h}-\sum_h\frac{Q_f^hG_f^h}{Q^hG^h}\sum_{i\neq h}\frac{P_i^hC_i^h}{Q_f^hG_f^h}\right)$$

Since $\sum_{i\neq h}\frac{P_i^hC_i^h}{Q_f^hG_f^h}=1$ according to Equation 37, and $N_i=\sum_i 1$ (defined as the number of foreign countries), it is implied that:

$$\theta\left(\sum_h\left(N-\chi_f^h\right)\right)=\gamma\left(\sum_h(N-1)\right)+\sigma\left(\sum_h\left(1-\chi_f^h\right)\right)$$

which can be rewritten (using $\chi_f^h=1-\chi_h^h$) as follows:

$$\theta=\gamma\Omega+\sigma\left(1-\Omega\right)\tag{40}$$

where $\Omega=\frac{\sum_h(N_i-1)}{\sum_h(N_i-1)+\sum_h\chi_h^h}$. This is the same expression as in Equation 18 in the main text.

Table 1 - Elasticities in Selected Studies

Paper	Trade Between	Elasticity
International Finance		
Backus, Kehoe, and Kydland (1994)	Home versus Foreign	1.5
Bergin (2006)	Home versus Foreign	1
Corsetti and Pesenti (2001)	Home versus Foreign	1
Corsetti, Dedola, and Leduc (2008)	Home versus Foreign	0.85
Heathcote and Perri (2002)	Home versus Foreign	0.9
Stockman and Tesar (1995)	Home versus Foreign	1
International Trade		
Anderson and Van Wincoop (2003)	Source Countries	$[5, 10]$
Anderson and Van Wincoop (2004)	Source Countries	$[5, 10]$
Clausing (2001)	Source Countries	9.6
Eaton and Kortum (2002)	Source Countries	9.28
Head and Ries (2001)	Source Countries	$[7.9, 11.4]$
Hummels (2001)	Source Countries	$[2.00, 5.26]$
Simonovska and Waugh (2014a)	Source Countries	$[3.79, 5.46]$
Simonovska and Waugh (2014b)	Source Countries	5.63

Notes: This is a very brief summary of studies selected among many others. For international trade studies, we have considered the elasticity measures at the macro (rather than the micro) level; see Simonovska and Waugh (2014b) for a nice discussion based on the difference between micro and macro elasticities in new international trade models.

Table 2 - Definition of Variables and Parameters in International Trade

Variables	Definition
C^h	Aggregate consumption in home country
C_i^h	Aggregate consumption of products from country i
C_{ij}^h	Consumption of good j imported from country i
P^h, P_i^h, P_{ij}^h	Prices per unit of C^h, C_i^h, C_{ij}^h, respectively
β_i^h, β_{ij}^h	Demand shifters of C_i^h, C_{ij}^h, respectively
$E^h = P^h C^h$	Total expenditure in home country
Parameters	
θ	Macro elasticity of substitution across all source countries
η	Elasticity of substitution across goods
$\varepsilon\left(\eta, \theta, \omega_{ij}^h, \omega_i^h\right)$	Price elasticity of demand for good j from country i
ω_i^h	Expenditure share of products from country i
ω_{ij}^h	Expenditure share of good j within country i products

Table 3 - Definition of Variables and Parameters in International Finance

Variables	Definition
G^h	Aggregate consumption in home country
C_h^h, G_f^h	Aggregate consumption of home and foreign products, respectively
C_i^h	Aggregate consumption of products from foreign country i
C_{ij}^h	Consumption of good j imported from foreign country i
$P_{ij}^h, P_i^h, P_h^h, Q_f^h, Q^h$	Prices per unit of $C_{ij}^h, C_i^h, C_h^h, G_f^h, G^h$, respectively
$1-\alpha_h^h, \alpha_h^h, \alpha_i^h, \beta_{ij}^h$	Demand shifters of $G_f^h, C_h^h, C_i^h, C_{ij}^h$, respectively
$E^h = Q^h G^h$	Total expenditure in home country
Parameters	
σ	Macro elasticity of substitution across home and foreign countries
γ	Elasticity of substitution across foreign countries
η	Elasticity of substitution across goods
$\varepsilon\left(\eta, \gamma, \sigma, \omega_{ij}^h, \chi_i^h, \chi_f^h\right)$	Price elasticity of demand for good j from foreign country i
χ_h^h	Expenditure share of home products
χ_f^h	Expenditure share of foreign products
χ_i^h	Expenditure share of products from foreign country i

Table 4 - Estimation Results

Estimated Coefficients			
	$\theta\delta$	$\gamma\delta$	$\sigma\delta$
Estimate	2.194	2.452	0.277
Lower Bound	2.106	2.310	0.017
Upper Bound	2.281	2.594	0.537
Sample Size	2346	2142	204
Adjusted R-Squared	0.559	0.439	0.949

Implied Elasticities when $\delta = 0.45$ with $\delta \in [0.18, 0.84]$			
	θ	γ	σ
Estimate	4.875	5.449	0.615
Lower Bound	2.508	2.750	0.020
Upper Bound	12.672	14.410	2.984

Notes: All regressions include source fixed effects, destination fixed effects, and quarter fixed effects, where observations are pooled across quarters. Lower and upper bounds represent the 95% confidence interval. The value of $\delta = 0.45$ (and the corresponding range between 0.18 and 0.84 across industries that is used in the calculation of lower and upper bounds for θ, γ and σ) is borrowed from Yilmazkuday (2012) as the average estimate across NAICS industries obtained by using interstate trade data (coming from Commodity Flow Survey) within the U.S. for the year of 2007. The estimations of θ, γ and σ have been achieved by dividing the estimates of $\theta\delta$, $\gamma\delta$ and $\sigma\delta$ with the average value of $\delta = 0.45$. The upper (lower) bounds of θ, γ and σ have been calculated by dividing the upper bounds of $\theta\delta$, $\gamma\delta$ and $\sigma\delta$ with the lower (upper) bound of $\delta = 0.18$ ($\delta = 0.84$).